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Fig. 1

Figure 1: Fig. 1

Abstract

Full Text

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ON A MODEL OF UNSTABLE COMBUSTION

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The magnitude of the acoustic impedance of a burning surface serves as a boundary condition in the problem of the stability of natural oscillations in theories of resonant combustion ([1–3]) and, to a considerable extent, determines the possibility of a stable combustion regime. However, experimental difficulties do not allow this quantity to be measured by standard acoustic methods. At the same time, the expressions for the impedance obtained from combustion theories ([4,5]) contain

Fig. 1

a certain arbitrariness owing to the different schematizations of the combustion process. Comparison of theory with experiment on the basis of conclusions following from ([1–3]) is made difficult by the fact that in papers ([1–3]) combustion is considered as taking place in a volume of complicated geometry, and the conditions at the exit from it are to a considerable degree arbitrary. This does not permit an unambiguous choice of an expression for the acoustic impedance of the burning surface. The difficulties that arise can be avoided if one considers a one-dimensional model in which instability can arise only on longitudinal acoustic modes (Fig. 1). The stability of combustion in such a system from the point of view of conservation of the gas balance was considered in ([6,7]). Acoustic instability of the indicated type is observed in the combustion of a condensed system in a tube, for example in so-called pyrotechnic whistles ([13]), which, when certain compositions are used in them—for example a mixture of 30% potassium benzoate and 70% potassium perchlorate—generate sound with a frequency approximately equal to the fundamental tone of the tube. The stability of a system analogous to the one considered here, in the case of gas combustion, was investigated in ([8]). It was also pointed out there that studying the properties of such a system opens up broad possibilities for investigating the nonstationary characteristics of combustion. In the case of combustion of condensed systems, such a complicating circumstance as curvature of the combustion front is absent.

Instability in a system of this type leads to the establishment of a self-oscillatory process whose frequency is the natural frequency of the system ([9,10]).

In the linear approximation, the problem of the stability of combustion of a condensed system in a tube can be formulated as an acoustic problem of natural oscillations in a tube closed by impedances Z_0 and Z_l. It is assumed here that the burning rate is small, so that the change in the length of the tube due to displacement of the combustion during the time of establishment of the oscilla-

perturbations can be neglected. The natural frequencies of such a system are determined from the equation [11]

$$\frac{Z_0+1}{Z_0-1} = \frac{Z_l-1}{Z_l+1} e^{-2ikl}, \tag{1}$$

where $k = \omega/c$ is the wave vector; Z_0 is the impedance of the burning surface; Z_l is the impedance of the open end of the tube.

The roots of equation (1) are, in general, complex, $\omega = \alpha + i\beta$. If $\beta < 0$, then the amplitude of the natural oscillation will increase with time. The value $\beta = 0$ corresponds to neutral oscillations and determines the stability boundary of the system. Starting from (1), the condition for self-excitation of the system can be written in the form

$$r_0 r_l > 1, \tag{2}$$

where $r_0 = |R_0|$, $R_0 = r_0 e^{2i\delta_0}$ is the reflection coefficient of the burning surface; $r_l = |R_l|$, $R_l = r_l e^{2i\delta_l}$ is the reflection coefficient of the open end of the tube. Condition (2) has a simple physical meaning: the decrease of the wave amplitude at the open end of the tube as a result of radiation, in the presence of instability, must be compensated by an increase in the wave amplitude upon reflection from the burning surface. At the stability boundary the equality $r_l = 1$ holds. If the impedances Z_0 and Z_l are known, then, using equation (1), one can find the values ω_{cr} and l_{cr} corresponding to the stability boundary. On the other hand, if the system has a region of instability, then, using the experimentally found values of the oscillation frequency ω_{cr} and the corresponding tube length l_{cr}, one can, with the aid of equation (1), determine the impedance of the burning surface $z_0(\omega_{\text{cr}}) = X_0(\omega_{\text{cr}}) + iY_0(\omega_{\text{cr}})$; for this it is also necessary to know the quantity Z_l. The impedance of the open end of a circular cylindrical tube Z_l was found in [12]. For $ka \ll 1$ it is equal to $Z_l = k^2a^2/4 + i \cdot 2ka/\pi$.

It should be noted that the possibility of using the expression for the acoustic impedance of the open end of a tube found in [12] is limited by a number of factors, such as the presence of a gas flow in the tube, the existence of a contact discontinuity separating the jet of gases flowing out of the tube from the surrounding gas, and the nonisentropic nature of the flow. Usually the influence of these factors is small and, to a first approximation, they may be neglected.

By varying the radius of the tube so that the criterion ensuring the presence in the system of only longitudinal acoustic modes is satisfied, and by recording the values ω_{cr} and l_{cr} for each value of the radius, one can determine the frequency dependence of the impedance of the burning surface $Z_0(\omega)$. It is usually assumed that the frequency of self-oscillations in such systems is a natural frequency of the system [9,10]. This means that, even far from the stability boundary, the relation $2\delta_0 + 2\delta_l - 2kl = 2\pi n$ remains valid. If it is assumed that δ_l does not depend on β, then this relation determines the frequency dependence $\delta_0(\alpha)$.

If the acoustic impedance of the burning surface does not depend on frequency and $r_0 > 1$, then the qualitative picture of the combustion process is as follows: while the tube length is less than l_{cr}, combustion is stable; as the condensed phase burns out, the natural frequency of the system decreases and finally reaches

$$\omega_{\text{cr}} = \frac{c}{a}\sqrt{1 - \frac{1}{r_0^2}},$$

Starting from this moment the process is unstable—the system generates sound oscillations. The situation will be different if the frequency dependence $Z_0(\omega)$ has the form predicted by the theory developed in [4]. In this case there must exist both upper and lower boundaries of unstable frequencies, so that the instability region occupies a certain frequency band. In this case there is also possible a situation in which the instability region altogether

is absent, despite the fact that for sufficiently large values of ω the reflection coefficient $r_0(\omega) > 1$.

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Note: Figure translations are in progress. See original paper for figures.