WIND-DRIVEN AND THERMOHALINE CIRCULATION IN THE OCEAN

1961

SovietRxiv

Abstract

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GEOPHYSICS

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(Presented by Academician V. V. Shuleikin, 18 I 1961)

The possibility of jointly taking into account the effects of wind and thermodynamic factors in the formation of currents in a deep baroclinic sea or ocean was indicated already in work ([1]). However, in subsequent works by the author and by other investigators, either only the wind-driven or only the thermohaline circulation in the sea was studied. Only in the work of Takano ([2]) was the superposition of wind-driven and convective currents considered under certain special conditions of zonal flow, with a constant value of the Coriolis parameter. Below, the same question is considered, concerning the formation of steady currents in a baroclinic ocean, without the indicated restrictions. Here the author develops the linearized treatment of the problem proposed by him earlier, justifying it to a considerable degree by the fact that the effect of turbulent diffusion of density is qualitatively equivalent to the effect of convective processes in the sea.

Let us consider separately the currents of the surface and deep layers of the sea ([3]). The currents of the surface layer satisfy the equations of Ekman theory

$$-2\omega\cos\theta\, v = \nu\frac{\partial^2 u}{\partial z^2} - \frac{1}{\rho}\frac{\partial p}{R\partial\theta}; \tag{1}$$

$$2\omega\cos\theta\, u = \nu\frac{\partial^2 v}{\partial z^2} - \frac{1}{\rho}\frac{\partial p}{R\sin\theta\,\partial\lambda}; \tag{2}$$

$$g\rho = \frac{\partial p}{\partial z}; \tag{3}$$

$$\frac{1}{R\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta\, u) + \frac{\partial v}{\partial\lambda}\right] + \frac{\partial w}{\partial z} = 0 \tag{4}$$

and the boundary conditions at the free surface: for $z = \zeta$,

$$\nu_0\rho_0\frac{\partial u}{\partial z} = -T_\theta, \qquad \nu_0\rho_0\frac{\partial v}{\partial z} = -T_\lambda, \qquad w = 0, \qquad p = 0 \tag{5}$$

and at the depth of the sea, as $z \to \infty$,

$$u = u_g = -\frac{1}{2\omega\cos\theta\,\rho}\,\frac{\partial p}{R\sin\theta\,\partial\lambda},$$

$$v = v_g = \frac{1}{2\omega\cos\theta\,\rho}\,\frac{\partial p}{R\partial\theta}. \tag{6}$$

Suppose that the density distribution ρ differs little from the known equilibrium distribution $\rho_*(z)$, so that

$$\rho = \rho_* + \rho', \qquad \rho' = \delta_0\delta(\theta,\lambda,z) + \sigma_0\sigma(\theta,\lambda,z), \tag{7}$$

where δ, σ are dimensionless quantities of the disturbances caused by the action of the wind and, respectively, by the action of nonuniform heat inflow, precipitation and evaporation, while δ_0, σ_0 are characteristic values of the same disturbances. From

from equation (3) we find the following expression for the deviation of the pressure p' from its equilibrium value $p_*(z)$:

$$p' = -g\rho_0\zeta + g\int_0^z (\delta_0\delta + \sigma_0\sigma)\,dz. \tag{8}$$

Estimating the order of magnitude of the quantities entering here, it is not difficult to show that for the surface layer of the sea the value of the integral may be neglected.

The purely drift components of the velocity satisfy equations (1), (2) without the last terms. They are determined by the well-known Ekman formulas.

Integrating the equation of continuity with respect to z from $z = \zeta$ to the lower boundary of the surface friction layer $z = h$, we find, taking into account (1), (2), (5), and (8), the value needed below of the vertical component of velocity w at $z = h$:

$$w_{z=h} = -\frac{1}{2\omega\rho}\,\mathrm{rot}_z\,\frac{T}{\cos\theta}. \tag{9}$$

The currents of the deep layer of the sea are gradient-convective and satisfy the geostrophic relations

$$-2\omega\cos\theta\,\rho v = -\frac{1}{R}\frac{\partial p}{\partial\theta},$$

$$2\omega\cos\theta\,\rho u = -\frac{1}{R\sin\theta}\frac{\partial p}{\partial\lambda}. \tag{10}$$

From the equation of continuity (4) there follows the relation

$$\frac{\partial w}{\partial z} = \frac{1}{2\omega\rho R^2\cos^2\theta}\frac{\partial p}{\partial\lambda}. \tag{11}$$

We use the linearized equation of turbulent diffusion for density (1)

$$w\frac{d\rho_*}{dz} = \varepsilon\nu_z\frac{\partial^2\rho}{\partial z^2}, \tag{12}$$

assuming here, as an approximation (5), that the effect of vertical turbulent exchange is predominant (in comparison with horizontal exchange). Further schematizing the problem, we replace $d\rho_*/dz$ by a certain value b, averaged over the depth of the baroclinic layer. Then differentiating both sides of (12) twice with respect to z and taking (11) into account, we arrive at an equation for ρ'

$$\frac{\partial\rho'}{\partial\lambda} = \frac{\partial^4\rho'}{\partial\xi^4}. \tag{13}$$

Here $\xi = -(z-h)/H$, where

$$H = \sqrt[4]{\frac{2\rho\omega R^2\cos^2\theta}{g}\,\frac{\varepsilon\nu_z}{b}}. \tag{14}$$

Let us state the boundary conditions for equation (13), referring them separately to δ and σ. At the upper boundary of the baroclinic layer, for $\xi = 0$, the vertical velocity must be continuous, and consequently, by (9) and (12),

$$\delta_0\frac{\partial^2\delta}{\partial\xi^2} = -\frac{bH^2}{2\rho\omega\varepsilon\nu_z}\,\mathrm{rot}_z\,\frac{T}{\cos\theta} = \delta_0 F(\theta,\lambda), \qquad \frac{\partial^2\sigma}{\partial\xi^2} = 0. \tag{15}$$

Suppose that the vertical component of the density gradient does not vary on the vertical segment $0 \le z \le h$, within the limits of the surface layer of the sea. Then, taking the density in the form of a linear function of temperature ϑ and salinity s:

$$\rho = \rho_0(1 - \alpha\vartheta + \beta s), \quad \alpha \sim 2\cdot10^{-4}\frac{1}{\mathrm{deg}}, \quad \beta \sim 8\cdot10^{-4}\frac{1}{{}^1/_{00}}, \tag{16}$$

one may assume that the value of $\partial\rho/\partial\xi$ at $\xi = 0$ is determined by the prescribed laws of distribution of the temperature gradient $\partial\vartheta/\partial z$, precipitation P, and evaporation E, or else by deviations from the equilibrium values of the heat flux $Q_0Q' = -\varepsilon v_0\partial\vartheta/\partial z$ and the fresh-water flux $\Pi_0\Pi' = P - E$ at the free surface of the sea. Thus, one may take (4), for $\xi = 0$, as

$$\frac{\partial \delta}{\partial \xi} = 0,$$

$$\sigma_0 \frac{\partial \sigma}{\partial \xi} = \frac{\rho_0 H}{\varepsilon v_\vartheta} \left(\frac{\alpha Q_0 Q'}{c \rho_0} + \beta s \Pi_0 \Pi' \right) = \sigma_0 M(\theta, \lambda). \tag{17}$$

In addition to conditions (15), (17), we require that the perturbations of density and pressure decay with depth: as $\xi \to \infty$,

$$\delta \to 0, \quad \sigma \to 0, \quad p', \left(\frac{\partial^3 \delta}{\partial \xi^3}, \frac{\partial^3 \sigma}{\partial \xi^3} \right) \to 0. \tag{18}$$

Let us also note the expression for δ_0 following from (9) and (11):

$$\delta_0 = \frac{T_0 R}{g H^2}. \tag{19}$$

Let the expansions of the prescribed functions $F(\theta, \lambda)$ and $M(\theta, \lambda)$ in series of the form be known:

$$F(\theta, \lambda) = \mathrm{Re} \sum_{n=1}^{\infty} a_n(\theta) e^{in\lambda},$$

$$M(\theta, \lambda) = \mathrm{Re} \sum_{n=1}^{\infty} c_n(\theta) e^{in\lambda}. \tag{20}$$

Then it is natural to seek the solution of the problem in the same form:

$$\delta(\theta, \lambda, \xi) = \mathrm{Re} \sum_{n=1}^{\infty} A_n(\theta, \xi) e^{in\lambda},$$

$$\sigma(\theta, \lambda, \xi) = \mathrm{Re} \sum_{n=1}^{\infty} C_n(\theta, \xi) e^{in\lambda}. \tag{21}$$

Equation (13) leads to two ordinary differential equations for $A_n(\xi)$, $C_n(\xi)$, solving which we find

$$A_n = -\frac{a_n}{\sqrt{2n}} \left(i e^{-k\xi} + e^{ik\xi} \right), \quad C_n = -\frac{c_{ne}^{-\pi i/8}}{\sqrt[4]{4n}} \left(e^{-k\xi} + e^{ik\xi} \right), \quad k = \sqrt[4]{n}\, e^{-\pi i/8}. \tag{22}$$

The case $n = 0$ corresponds to a purely zonal flow and is of no physical interest. It is interesting to note that the dependence $H \sim \sqrt{\cos\theta}$ according to (14) is qualitatively similar to that found earlier by the author ([1]) and by other investigators ([6,7]) ($H \sim \cos\theta$), despite the differences in the initial assumptions in solving the problem posed.

Let us take $\varepsilon\nu_z = 1$ cm^2/sec, $b = 10^{-8}$ g/cm^4, $T_0 = 1$ g/cm $\cdot$ sec^2. Then, from (19), $\delta_0 \sim 2.6 \cdot 10^{-4}$ g/cm^3. Next let $\Pi_0 = 5 \cdot 10^{-7}$ cm/sec, $Q_0 = 10^{-4}$ cal/cm$^2 \cdot$sec, $\varepsilon\nu_0 \sim 10$ cm^2/sec, $s_0 = 30‰$, $L = 6 \cdot 10^8$ cm. From these data we find from (17) that $\sigma_0 \sim 2 \cdot 10^{-4}$ g/cm^3. Thus, the characteristic values of the wind and thermohaline perturbations in the density field are of the same order.

From the expressions found for δ and σ, ρ', p', and ζ are determined directly, and then also the velocity components of the gradient-convective currents in the ocean.

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Received
16 I 1961

CITED LITERATURE

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Note: Figure translations are in progress. See original paper for figures.