

# ON THE POSSIBILITY OF GENERALIZING THE PRINCIPLE OF INVARIANCE IN LINEAR AUTOMATIC CONTROL SYSTEMS WITH VARIABLE PARAMETERS

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**Abstract**

**Full Text**

**CYBERNETICS AND CONTROL THEORY**

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**ON THE POSSIBILITY OF GENERALIZING  
THE PRINCIPLE OF INVARIANCE IN LIN-  
EAR AUTOMATIC CONTROL SYSTEMS  
WITH VARIABLE PARAMETERS**

*(Presented by Academician V. S. Kulebakin, 12 II 1961)*

In the well-known works of N. N. Luzin and P. I. Kuznetsov <sup>(1-3)</sup>, the theory of the independence of any coordinate of a linear dynamic system from external actions was mathematically substantiated. The subsequent development of this idea by V. S. Kulebakin <sup>(4,5)</sup> made it possible to create a new direction in the theory of automatic control and to formulate the basic laws governing combined automated systems.

However, the implementation of the principle of invariance, developed for systems with constant parameters, in electrical systems gives rise in a number of cases to certain difficulties, since the elements of the circuit often change their parameters both with increasing operating time and over shorter intervals under the influence of fluctuations in temperature, humidity, etc. Similar phenomena also occur in other physical objects, which necessitates the use, in systems with variable parameters, of special complex correcting devices for the continuous adjustment of the parameters of the disturbance-action circuit in accordance with changes in the characteristics of the regulator and the object <sup>(6)</sup>.

At the same time, the basic propositions of the theory of invariance can be extended also to systems with variable parameters if disturbances are understood to mean not only external actions applied to a linear dynamic system, but also internal disturbances caused by changes in the parameters of the system over time or by the discrepancy of their nominal values from the required value. In this case, on the basis of the theory of invariance, it is possible to solve the problem of constructing optimal (according to some criterion) linear automatic control systems. For this it is sufficient to know the required (optimal) linear characteristic of the controlled object, and to regard deviations from it in the real system as the result of the action of external disturbances, with respect to which the requirements of independence of the input (controlled) coordinate must be satisfied.

Let a system of linear differential equations be given, describing the dynamic system under study:

$$\begin{aligned}
 a_{11}x_1 + a_{12}x_2 + \dots + a_{1l}x_l + \dots + a_{1n}x_n &= f_1(t), \\
 a_{21}x_1 + a_{22}x_2 + \dots + a_{2l}x_l + \dots + a_{2n}x_n &= f_2(t), \\
 &\dots \\
 a_{k1}x_1 + a_{k2}x_2 + \dots + a_{kl}^*x_l + \dots + a_{kn}x_n &= f_k(t), \\
 &\dots \\
 a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nl}x_l + \dots + a_{nn}x_n &= f_n(t),
 \end{aligned} \tag{1}$$

where  $a_{ij}$  is an operational polynomial with constant coefficients of degree not higher than the second ( $i \neq k$  for  $j = l$ ),  $a_{kl}^*$  is an operational polynomial with variable coefficients.

with variable parameters,

$$a_{kl}^*x_l = b_0(t)x_l'' + b_1(t)x_l' + b_2(t)x_l, \quad x_j$$

are unknown functions of time.

Suppose that the system becomes optimal (according to some criterion) when

$$a_{kl}^* = a_{kl}.$$

Here

$$a_{kl}x_l = c_0x_l'' + c_1x_l' + c_2x_l,$$

where  $c_0$ ,  $c_1$ , and  $c_2$  are constant coefficients.

Let us determine the conditions under which some coordinate  $x_j$  of system (1) becomes independent of one or several external forces  $f_i(t)$  and of the deviations of the parameters  $b_0(t)$ ,  $b_1(t)$ , and  $b_2(t)$  from the values  $c_0$ ,  $c_1$ , and  $c_2$ , respectively. We shall assume that the disturbance  $f_k(t)$  belongs to the class of analytic functions holomorphic at the point  $t = t_0$ .

Let us rewrite the system of equations (1) in the following form:

$$\begin{aligned}
 a_{11}x_1 + a_{12}x_2 + \dots + a_{1l}x_l + \dots + a_{1n}x_n &= f_1(t), \\
 a_{21}x_1 + a_{22}x_2 + \dots + a_{2l}x_l + \dots + a_{2n}x_n &= f_2(t), \\
 &\dots \\
 a_{k1}x_1 + a_{k2}x_2 + \dots + a_{kl}x_l + \dots + a_{kn}x_n &= F_k(t), \\
 &\dots \\
 a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nl}x_l + \dots + a_{nn}x_n &= f_n(t),
 \end{aligned} \tag{2}$$

where  $F_k(t) = f_k(t) + a_{kl}x_l - a_{kl}^*x_l$  is the  $k$ -th resultant disturbance.

Fig. 1. System with regulation by disturbance, 1, 2—elements of the regulated variable and the model with optimal dynamic characteristics, 4—solving device for determining the resultant reduced disturbance

Figure 1: Fig. 1. System with regulation by disturbance, 1, 2—elements of the regulated variable and the model with optimal dynamic characteristics, 4—solving device for determining the resultant reduced disturbance

To satisfy the stated requirement, since system (2), in contrast to system (1), is a system with constant coefficients, a necessary and sufficient condition is <sup>(1)</sup>

$$\Delta^{ij} = 0, \quad (3)$$

where  $\Delta_{ij}$  are the corresponding minors of the determinant of system (2), identically nonzero,

$$|a_{ij}| \neq 0.$$

On the basis of the first fundamental theorem of Luzin–Kuznetsov <sup>(1)</sup>, the following conclusion can be drawn:

If it is possible to indicate such a differential operator  $a_{kl}$  that transforms the function  $F_k(t)$  into a function holomorphic on some interval  $(a, b)$  containing the point  $t_0$ , and if conditions (3) are satisfied and system (2) admits a solution  $(x_1, x_2, \dots, x_n)$  under zero initial conditions  $(x_j(t_0) = x'_j(t_0) = 0)$ , then any coordinate  $x_j$  of system (1) is independent of changes in the parameters  $b_0(t) - c_0$ ,  $b_1(t) - c_1$ , and  $b_2(t) - c_2$  over the entire interval  $(a, b)$ .

**Fig. 1.** System with regulation by disturbance; 1, 2—elements of the regulated variable and of the model with optimal dynamic characteristics; 3—model with optimal dynamic characteristics; 4—solving device for determining the resultant reduced disturbance.

This means that the invariance principle in the present case can also be extended to linear systems with variable parameters.

In practice, the problem reduces to finding the resultant disturbance  $F_k(t)$  with respect to which the requirements of independence of the regulated coordinate must be fulfilled. The function  $F_k(t)$  can be determined by comparing the response of the system under study (1) to the application of the inputs  $f_1, f_2, \dots, f_n$  and the response of the model of the optimal system with constant parameters to the control input.

For simplicity of notation, suppose that the system is acted upon only by the control action  $f_1(t)$  and the disturbing action  $f_k(t)$ . In accordance with (2), system (1) may be regarded as optimal, with, in addition to the control  $f_1(t)$ , the disturbing signal  $F_k(t)$  applied to it. This means that, in order for the

regulated coordinate  $x_j$  to vary according to the optimal law (which is specified by a model connected in parallel with the system under study and acted upon only by the control signal), it is necessary and sufficient to apply to the input of the system an action equal to the resultant disturbance, taken with the opposite sign and referred to the point of application of the signal  $f_1(t)$ :

$$-\varphi_k(t) = -A_3 F_k(t),$$

where  $A_3$  is the operator which refers  $F_k(t)$  to the input of the system.

It is not difficult to show that  $\varphi_k(t)$  is computed as follows:

$$\varphi_k(t) = A_1^{-1} [A_2 f_1(t) + A_2 \psi_k(t) - A_1 f_1(t)],$$

where  $A_1$  and  $A_2$  are the operators determining the relation between the output and input coordinates of the model and of the system, respectively;  $\psi_k(t) = A_3 f_k(t)$  is the disturbance referred to the input of the system;  $A_2 f_1(t) + A_1 \psi_k(t)$  is the response of the system to the actions  $f_1$  and  $f_k$ ; and  $A_1 f_1(t)$  is the response of the model to the control signal  $f_1$ .

A structural diagram of an invariant system of this type is shown in Fig. 1.

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*Note: Figure translations are in progress. See original paper for figures.*

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