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Abstract

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PHYSICAL CHEMISTRY

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ON THE THEORY OF EROSIVE BURNING OF PROPELLANTS

(Presented by Academician V. N. Kondrat' ev on 2 VII 1960)

It is known ([1–3]) that a turbulent gas flow passing over a burning surface noticeably increases the burning rate of a propellant. In the present note a theory of this question is developed, based on the theory of stationary burning ([4]).

Let us consider a semi-infinite plate of burning propellant. Suppose that far from the leading edge an asymptotic regime of gas flow is established. The system of equations describing this case is as follows:

$$\overline{\rho v} = m_t = \text{const}, \qquad p = \text{const}; \tag{1}$$

$$\overline{\rho v}\,\frac{du}{dy} = \frac{d}{dy}\left[(\mu + \mu_t)\,\frac{du}{dy}\right]; \tag{2}$$

$$c\,\overline{\rho v}\,\frac{d\overline{T}}{dy} = \frac{d}{dy}\left[(\mu + \mu_t)\,\frac{d\overline{T}}{dy}\right] + Qf(\overline{a}, \overline{T}); \tag{3}$$

$$\overline{\rho v}\,\frac{d\overline{a}}{dy} = \frac{d}{dy}\left[(\mu + \mu_t)\,\frac{d\overline{a}}{dy}\right] - f(\overline{a}, \overline{T}). \tag{4}$$

Notation: m_t is the mass burning rate in the turbulent flow; ρ is density; $\overline{v}$ is the projection of the velocity on the y-axis; u is the projection of the velocity on the x-axis; μ_t is the coefficient of "turbulent"dynamic viscosity; $\overline{T}$ is temperature; $\overline{a}$ is the relative concentration of the reacting substance; Q is the heat effect; $f(\overline{a}, \overline{T})$ is the total rate of the chemical reaction; c is heat capacity. The remaining notation coincides with that of work ([4]).

In writing the system (2)–(4), we have assumed that

$$c\mu = \lambda; \qquad \nu = D; \qquad c\mu_t = \lambda_t; \qquad \nu_t = D_t, \tag{5}$$

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where λ, D, ν are, respectively, the molecular coefficients of thermal conductivity, diffusion, and kinematic viscosity, while λ_t, D_t, ν_t are the coefficients of "turbulent" thermal conductivity, diffusion, and kinematic viscosity. The assumptions (5) are generally accepted. In addition, in equation (3) we neglect the heat of friction in comparison with the heat arising as a result of the progress of the chemical reaction.

The system of equations (3) and (4) has a first integral *

$$\frac{\overline{a}}{\overline{a}_{s1}} = \frac{\overline{T}_{11} - \overline{T}}{\overline{T}_{11} - \overline{T}_{su}}, \tag{6}$$

and, consequently, is reduced to a single equation

$$\lambda \frac{d}{dy}\left[\left(1 + \frac{\nu_t}{\nu}\right)\frac{d\overline{T}}{dy}\right] - cm_t\frac{d\overline{T}}{dy} + Q_\beta f_\beta(\overline{T}) = 0. \tag{7}$$

* In what follows only the α- and β-stages are considered ([4]).

α-stage. Assuming that the turbulent pulsations do not penetrate into the liquid-viscous layer of the k-phase, for the α-stage we shall have the usual system of equations. Therefore

$$n_t = B_{\alpha t} e^{-E_\alpha/2R\overline{T}_{su}}. \tag{8}$$

Here the surface temperature $\overline{T}_s$ of the k-phase is marked by the subscript u, which indicates that this temperature depends parametrically on the velocity of the core of the flow.

It is not difficult to verify that the erosion ratio $\mathcal{E}$ is equal to

$$\mathcal{E} = \frac{m_t}{m} = \exp\left[\frac{E_\alpha}{2R}\left(\frac{1}{\overline{T}_s} - \frac{1}{\overline{T}_{su}}\right)\right]. \tag{9}$$

β-stage. Integrating equation (3), we obtain

$$\sigma_s = (\mu + \mu_t^r)\frac{d\overline{u}}{dy} - m_t\overline{u}, \tag{10}$$

where σ_s is the tangential friction stress on the burning surface of the k-phase. Equation (10) may be replaced by the simpler one:

$$\frac{\sigma_s}{\rho_{11}} = (\nu_1 + \nu_{t1})\frac{du}{dy}; \qquad \nu_1 = \frac{\mu}{\rho_{11}}; \qquad \nu_{t1} = \frac{\mu_t}{\rho_{11}}, \tag{11}$$

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in which the temperature drop and the inflow of gas mass from the burning surface of the k-phase are not taken into account.

The accepted assumption is not a crude one and has a simple physical explanation. Indeed, from boundary-layer theory it is known that the inflow of gas mass from a porous plate leads to a noticeable decrease in frictional resistance; on the other hand, the cooling effect of a wall washed by a gas stream leads to a noticeable increase in frictional resistance. Thus, if both factors occur simultaneously—the inflow of mass and the heat flux from the gas to the wall (as in the combustion of a powder plate)—then they partially compensate each other.

Introducing the dynamic velocity $u_\tau = \sqrt{\sigma_s/\rho_{11}}$ and the dimensionless distance from the wall $y^* = u_\tau y/\nu_1$, we find

$$\frac{d(\overline{u}/u_\tau)}{dy^*} = (1 + \nu_{t1}/\nu_1)^{-1}. \tag{12}$$

Equation (12) makes it possible, from a known velocity profile, to find the coefficient of turbulent viscosity ν_{t1}. In work ([5]) it is shown that for $0 \le y^* \le 30$ the velocity distribution obeys the law

$$\frac{\overline{u}}{u_\tau} = \frac{1}{\sqrt{k_1}} \operatorname{th} \sqrt{k_1}\, y^*, \tag{13}$$

where k_1 is a constant coefficient; therefore,

$$\frac{\nu_{t1}}{\nu_1} = \operatorname{sh}^2 \sqrt{k_1}\, y^*. \tag{14}$$

Using (14), we easily find the temperature distribution in the preheating zone of the β-stage:

$$\ln \frac{\overline{T} - \overline{T}_{s1}}{\overline{T}_{su} - \overline{T}_{s1}} = \frac{cm_t}{\lambda} \frac{\nu_1}{\sqrt{k_1} u_\tau} \operatorname{th} \sqrt{k_1}\, y_u^*. \tag{15}$$

In particular, for small $\sqrt{k_1} y_u^*$, $\overline{T}_{su} \to \overline{T}_s$, one obtains the usual temperature profile

$$\ln \frac{\overline{T} - \overline{T}_{s1}}{\overline{T}_s - \overline{T}_{s1}} = \frac{cm}{\lambda} \frac{\nu_1}{\sqrt{k_1} u_\tau} \sqrt{k_1}\, y_0^*. \tag{16}$$

For the chemical-reaction zone of the β-stage, the simplified equation is valid

$$\lambda \frac{d}{dy} \left[\left(1 + \frac{\nu_{t1}}{\nu_1} \right) \frac{d\bar{T}}{dy} \right] = -Q_\beta t_\beta(\bar{T}). \tag{17}$$

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Remaining within the assumptions used by Ya. B. Zeldovich and D. A. Frank-Kamenetskii [6], it can be shown that

$$m_t^2 = \left(1 + \frac{\nu_{t1}}{\nu_1}\right) \frac{2\lambda}{c(\bar{T}_{11} - \bar{T}_{s1})} \int_{\bar{T}_{su}}^{\bar{T}_{11}} f_\beta(\bar{T})\, d\bar{T}, \tag{18}$$

where ν_{t1}/ν_1 is evaluated at the point corresponding to the highest temperature of the β-stage of the combustion process.

Using (18) and the analogous formula from the theory of steady combustion [4], we find:

$$\frac{m_t}{m} = \sqrt{\left(\frac{\bar{T}_{11} - \bar{T}_s}{\bar{T}_{11} - \bar{T}_{su}}\right)^{\nu_\beta}} \operatorname{ch}\left(\sqrt{k_1}\, y_{u1}^*\right), \tag{19}$$

where y_{u1}^* is evaluated at the point $\bar{T}_{11}$. Eliminating y_{u1}^* in (19), we obtain

$$\left(\frac{m_t}{m}\right)^2 = \frac{\bar{T}_{11} - \bar{T}_s}{\bar{T}_{11} - \bar{T}_{su}} + \frac{\sqrt{k_1}}{2\sqrt{2}} \ln\left(\frac{\bar{T}_{11} - \bar{T}_{s1}}{\bar{T}_{su} - \bar{T}_{s1}}\right) \frac{\rho_{11} u_{\text{cp}}}{m} \sqrt{\lambda}\,, \tag{20}$$

where u_{cp} is the flow velocity averaged over the cross section; λ is the drag coefficient, which in a first approximation can be found from expression (7)

$$\lambda = 0.0032 + \frac{0.221}{\text{Re}^{0.23}}. \tag{21}$$

The formulas obtained, (9) and (20), in principle solve the problem of erosive burning of propellants.

Figure 1 presents the results of a calculation example for nitroglycerin propellant at $\text{Re} = 10^6$ for various pressures, and experimental points for this propellant borrowed from work [3]. As follows from the graph, the erosive ratio $\mathcal{E}$ depends strongly on the gas-flow velocity and pressure. If the erosive ratio is represented not as a function of the gas-flow velocity, but of the dimensionless complex

$$J = \frac{\rho_{11} u_{\text{cp}}}{m} \sqrt{\lambda}\,,$$

then, probably, many experimental results for different grades of fuels can be reduced to a single universal curve.

Fig. 1. 1 –$p = 19.6$ atm.; 2 –44.0 atm.
3 –71.2 atm.

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Note: Figure translations are in progress. See original paper for figures.

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