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Abstract

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THEORY OF ELASTICITY

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POSTCRITICAL DEFORMATIONS OF CYLINDRICAL SHELLS UNDER EXTERNAL PRESSURE

This note presents some results of a study of elastic postcritical deformations of cylindrical shells, hinged along the edges, under the action of external pressure uniformly distributed over the surface. By definition, we call postcritical those deformations in which the shape of the shell differs substantially from the initial one; in other words, the deflections of the shell are comparable with its geometrical dimensions. The investigation of elastic postcritical states of a shell lies outside the range of applicability of the linear theory. At the same time, the study of such states is of unquestionable theoretical and practical interest.

The point is that the failure of a thin shell under external pressure is connected primarily with postcritical deformations resulting from loss of stability of the basic, cylindrical form. Therefore, in designing shells, it is natural to take as the design load such a load as would not lead to loss of stability. However, this simple and natural solution of the problem encounters serious difficulties, since the value of the critical load q_e, at which loss of stability occurs, is subject to a strong and rather uncertain influence of various factors, especially the initial deflection. This is explained by the instability of the equilibrium state of the shell under the action of the critical load q_e.

The load borne by the shell during the transition to postcritical deformations as a result of loss of stability decreases, reaches a certain minimum value q_i, and then begins to increase. The quantity q_i is called the lower critical load. In contrast to q_e, which is usually called the upper critical load, the lower critical load q_i is more stable. If the load q_i is taken as the design load, then the transition of the shell to postcritical deformations will be completely excluded. The lower critical load has been investigated theoretically and experimentally by many authors (see ([1])).

The basis of the method of investigation used by us is a way of approximating the shape of the shell in postcritical elastic equilibrium. First of all, we proceed from the assumption that the postcritical deformation is mainly a geometrical

Fig. 1

Figure 1: Fig. 1

bending. This assumption is natural, since the internal deformations of the middle surface of the shell, as elastic deformations, are small. Further, we assume that the postcritical deformation is a development of the buckling pattern on the surface of the shell at the moment of loss of stability.

These two considerations are essentially sufficient to reproduce the shape of the shell under postcritical deformation by a surface Z isometric to the cylinder, to within a certain function of one variable. We determine this function from energy considerations, namely from the condition of minimum energy of elastic deformation of the shell into the form Z for a fixed work A performed by the external load.

The surface Z consists of $2n$ common cylindrical regions Z_k with generators perpendicular to the axis of the surface (Fig. 1). Adjacent regions Z_k are separated by edges γ_k. In determining the elastic strain energy of the shell in the form Z, we divide it into two parts, U_γ and U_z, similarly to how this was done in the author' s papers ([2,3]). U_γ is the energy of strong local bending along the edges γ_k, while U_z is the bending energy over the main surface of the shell. The energy U_z is determined in the usual way, and the energy U_γ by means of the formula obtained in ([2]).

Fig. 1

Minimizing the elastic strain energy for fixed work A done by the external load, we find the shell shape Z and the energy U as functions of the parameter λ, which characterizes the overall deformation. The load q taken up by the shell for a given deformation λ is determined from the equilibrium condition $dU(\lambda) = dA(q, \lambda)$.

We proceed to present the results. First of all, we divide all cylindrical shells into three classes, depending on the geometrical dimensions and the elastic characteristics of the material. This division into classes is carried out by means of two conditions.

Condition A. $R\delta/L^2 \leqslant 1$ (δ is the shell thickness, R the radius, L the length).

Condition B. $0.4\,E\,(\delta/R)(R\delta/L^2)^{-1/3} < \sigma_{\text{v}}$ (E is the elastic modulus of the shell material, σ_{v} is the temporary strength).

We assign shells to the first class if condition A is not satisfied for them. Such shells are not considered by us.

For the second class of shells, which we call relatively thick, condition A is satisfied, but condition B is not. The transition to postcritical deformations in such shells is inevitably associated with the appearance of plastic deformations in the material. A qualitative investigation of the question of postcritical deformations of thick shells leads to the conclusion that their lower critical load q_i is close to the upper q_e.

We call shells comparatively thin if they satisfy both conditions A and B. For thin shells, the postcritical equilibrium states are elastic. It is shown that the equilibrium states of a thin shell in the first stage of postcritical deformation, which begins with the loss of stability of the cylindrical form, are unstable. As the deformation increases, the load taken up by the shell decreases. When the edges on the shell surface in the postcritical state come sufficiently close together, the load taken up by the shell increases. The second stage of postcritical deformation begins, in which the equilibrium states are stable.

If the external pressure q on the shell is characterized by the dimensionless quantity $\bar{q} = qR^2/E\delta^2$, then for the value $\bar{q}_i^0$ of the smallest load taken up by the shell, i.e., the lower critical load, the formula obtained is

$$\bar{q}_i^0 = \bar{q}_e^0 \left(2\varepsilon^{1/3} + 1.5\varepsilon^{1/2}\right), \qquad \varepsilon = R\delta/L^2,$$

where $\bar{q}_e^0 = 0.86\varepsilon^{1/2}$ is the upper critical load.

The question of the magnitude of the lower critical pressure $\bar{q}_i$ on a cylindrical shell in the presence of a small axial compression p is also considered. Here the method of investigation remains the same, and the result obtained is as follows. If the axial compression p is characterized by the dimensionless quantity

$\bar{p} = pR/E\delta$, then for the value of the lower critical pressure in the presence of axial compression, in the case of thin shells, one obtains the formula

$$\bar{q}_i = \bar{q}_i^0 - 7.75\varepsilon^{1/2}\bar{p},$$

where $\bar{q}_i^0$ is the dimensionless critical pressure in the absence of axial compression.

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Note: Figure translations are in progress. See original paper for figures.