CYBERNETICS AND CONTROL THEORY

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Abstract

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ON IMPROVING THE QUALITY OF EXTREMAL CONTROL OF INERTIAL OBJECTS IN THE PRESENCE OF DISTURBANCES

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Solving the problem of extremal control of inertial objects in the presence of low-frequency external disturbances requires the use of special systems ([1,2]). In cases where extremal controllers of the usual type are used, for example with memorization of the extremum, the influence of the object' s inertia makes it necessary to reduce the search speeds, which, in turn, worsens the disturbance immunity of the system.

In the work ([2]) a method of extremal control is proposed and theoretically substantiated which makes it possible, in principle, to eliminate the harmful influence of inertia of an object of any order in the absence of dynamic links in the object located before its extremal characteristic. The indicated system is distinguished by the fact that a certain, generally nonlinear, combination of signals corresponding to derivatives of the controlled quantity is applied to the input of the controller; moreover, by means of a nonlinear element an action changing the function is introduced.

The system has better properties with respect to the quality of control processes of inertial objects under low-frequency disturbances even in comparison with a usual-type system with a non-inertial object, since the effects of low-frequency disturbances are to a certain extent filtered out. However, in the presence of strong disturbances that rapidly change the level of the extremum of the function, and when the actuating member is not readjusted very quickly, the losses in the search may nevertheless be considerable.

Let us consider the frequently encountered case of extremal control of a first-order object. Let the characteristic of the extremal element have the form

$$y_1 = -K_x x^2, \tag{1}$$

where x and y_1 are respectively the coordinates of the input and output of the extremal element (e.e.). Let also $|\dot{x}| = V_x = \text{const}$, and let the dependence of the object output y_2 on y_1 have the form

Fig. 1

Figure 1: Fig. 1

$$y_2 = \frac{k_0}{p\tau_0 + 1}\, y_1, \tag{2}$$

where k_0 and τ_0 are constants.

According to the above-mentioned method of control ([2]), if the action of the nonlinear element is not taken into account, in the present case a signal $z = K_d y_2$ is applied to the input of the extremal controller. Denoting $y_1 K_d K_0/\tau = y$, we obtain

$$z = \frac{p\tau_0}{p\tau_0 + 1}\, y. \tag{3}$$

The transient processes in the control system under consideration are described by a system of equations analogous to that which describes transient processes in a non-inertial object with the use of a dynamic converter of the input signal.

The dynamics of an extremal-control system with a dynamic converter of the form (3) was considered in works ([3,4]), in which, in order to determine the parameters of transient processes and limit cycles with allowance for external disturbances, a method was used for constructing the phase trajectories of the system on a multisheet surface ([5]) in the coordinates $\alpha = x/V_x\tau_0$, $\beta = \dot{z}/2K_x V_x^2 \tau_0$.

Fig. 1

Let us apply this method in the present case. Within the intervals between reversals of the e.o. (executive organ) we have

$$\beta = (\beta\ + 1)e^{-|\Delta\alpha|} - 1,$$

where $\Delta\alpha = \alpha - \alpha$; β and α are the coordinates of the representative point at $t = 0$. Reversals of the e.o. occur each time after the curve β crosses the abscissa axis and the change in the area between the curve β and the abscissa axis reaches the value $S\ = \delta\ /2\eta\nu_{\tau_0}^2$, where $\delta\ = z\ /y$, z is the dead zone of the regulator, y is the value of the controlled quantity at the extremum point, $\eta = K_x x_{\max}^2/y$, $\nu_{\tau_0} = \tau_0 V_x/x_{\max}$. At the reversal points β changes abruptly by $\pm 2\alpha$, depending on the value of $\dot{\alpha}$.

Suppose that, starting from some time $t = 0$, at $x = 0$ and $y = 0$, the action of the external disturbance is expressed by a linear function of time. Then instead of (1) we have

$$y_1 = -K_x x^2 + K_1 t, \tag{4}$$

Fig. 2

Figure 2: Fig. 2

and at the initial instant (at $t = 0$) $\beta = \beta_0 = K_1/2K_xV_x^2\tau_0$. The control process corresponding to this case, in projection onto the phase plane α, β, for the system parameters $\delta = 0.5\%$, $\lambda_1 = K_1/K_xV_xX_{\max} = 1$,

$\nu_{\tau_0} = 5$ is shown in Fig. 1. As can be seen, oscillations with decreasing amplitude occur around the extremum; these contract to a limiting cycle with amplitude $\alpha = \alpha_p$. For the indicated parameters, the change in α during the oscillation process is equal to -0.11 (which corresponds to $\psi = x/x_{\max} = 0.55$), and the settling time is $\simeq 3\tau_0$. The introduction of a commutator that periodically reverses the system after a time t_k reduces the indicated deviations to the value $\psi_k = V_xt_k/x_{\max}$.

In the presence of a strong disturbance whose action is a quadratic function of time, which gives $y_1 = -K_xx^2 + K_vt^2$, the deviations of the system from the extremum in the limiting cycle, in the absence of a dead zone in the regulator, are estimated by the simplified formula

$$\psi_p = -\frac{\nu_{\tau_0}}{1.15}\ln(1 - \lambda_v), \tag{5}$$

where $\lambda_v = K_v/K_xV_x^2$. These deviations, for a large value of τ_0 and a limited velocity V_x, may be significant. Thus, for example, when $\lambda_v = 0.2$ and $\nu_{\tau_0} = 5$, $\psi_p = 1$. When the relay time of the actuator is decreased, the duration of the transient process and the amplitude $\Delta\alpha$ decrease both in the case of a linear and in the case of a quadratic disturbance.

Fig. 2

The quality of extremal control of inertial objects in the presence of disturbances can be improved by applying the combined system for transforming the input signal shown in Fig. 2.

The structural diagram of the system includes the following elements: e. e.—an extremal element with characteristic $y_1 = -K_xx^2$; o.—the dynamic part of the object, described by the differential equation $y_2^n + \psi_1(y_2', y_2'', \dots, y^{(n-1)}) + \psi_2(y_2) = k_0y_1$; y. f. s.—a shaping device producing the signal y, a dynamic converter and an extremal regulator e. r. Thus, the system contains the device described in work ([2]), forming a signal from the derivatives and the change of the function y_2, which eliminates the influence of the inertia of the controlled object and, in addition, it has a dynamic converter, which is a proportional element covered by integrating feedback with a dead zone z_0. The dynamic converter, at an optimal value of its time constant τ, in a number of cases practically eliminates or reduces the unfavorable influence of low-frequency disturbances. Introducing the dead zone z_0 into the feedback increases the accuracy of control

Fig. 3

Figure 3: Fig. 3

on the flat sections of the extremal characteristic near the extremum under the action of monotonic external disturbances changing x, and at a low velocity of the actuator. This system makes it possible to greatly increase the sensitivity of the regulator ([6]).

For determining the parameters of transient processes and limiting cycles of the system in this case, it is also convenient to apply the method of constructing its phase trajectories in the coordinates β and α on a multisheet surface.

In the ideal case—when there are no instrumental distortions in the operation of the device eliminating the influence of the object' s inertia—it may be assumed that $y = k_d y_1$.

The transformation of the signal y into $\dot{z} = u$ is determined by the equations

$$u = \frac{p^2\tau}{p\tau + 1}\, y \quad \text{for } -z_0 > z > z_0; \tag{6}$$

$$u = py \quad \text{for } -z_0 < z < z_0. \tag{7}$$

The transition from equation (6) to equation (7), in the absence of external disturbances, is made when $\dot{y}_k = 0$; in this case $\dot{z} = \dot{y}$, which corresponds to $\beta = \beta_\text{p} = \alpha$.

In the presence of an external disturbance that is a linear function of time,

Fig. 3

$$\beta_\text{p} = \lambda_1 / 2v_\tau - \alpha \operatorname{sign} \dot{\alpha},$$

where

$$v_\tau = \tau V_x / x_\text{max}.$$

Operation of the system within the dead zone of the feedback z_0 corresponds to a certain change in β_p, for which the area between the indicated segment of the straight line β_p and its projection onto the abscissa axis is equal to $S_0 = \delta_0 / 2\eta v_\tau^2$, where $\delta_0 = z_0 / y$. The phase trajectories corresponding to (6) have the same form as for equation (3).

Figure 3 gives, in projection onto the phase plane, the phase trajectories of the extremal-control system for $v_\tau = 0.15$ and for the same external conditions as in the case corresponding to Fig. 1. As is seen from Fig. 3, the deviations of the controlled object from the position corresponding to the extremum of the function have decreased, and the process of establishing the limiting cycle has been considerably accelerated; in this case, in the transient regime ψ does not exceed 0.33.

In the case of external disturbances that are a quadratic function of time, the control accuracy is likewise determined by formula (5), but in this case v_τ is now determined by the ratio $\tau V_x/x_{\max}$, which may be many times smaller than $\tau_0 V_x/x_{\max}$. Accordingly, the magnitude ψ_{p} is reduced. The introduction of a commutator into the control system makes it possible to greatly reduce the indicated short-time deviations of the system.

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Note: Figure translations are in progress. See original paper for figures.