

# MOTION OF A MEDIUM WITH FINITE CONDUCTIVITY IN THE PRESENCE OF A PLANE MAGNETIC FIELD

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**Abstract**

**Full Text**

**MATHEMATICAL PHYSICS**

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**MOTION OF A MEDIUM WITH FINITE CONDUCTIVITY IN THE PRESENCE OF A PLANE MAGNETIC FIELD**

*(Presented by Academician N. N. Bogolyubov, 4 II 1961)*

In studying the flow of an electrically conducting medium between solid boundaries, it was assumed that all parameters characterizing the flow of the medium are unchanged in the direction of the flow <sup>(1, 2)</sup>. As was shown in <sup>(3, 4)</sup>, one can find such plane magnetic fields of the form  $\mathbf{H} = [a(y) - bx]\mathbf{i} + (by + b_0)\mathbf{j}$ , for which motion of the medium is possible with velocity  $v_x = u(y)$ ,  $v_y = v_z = 0$ . In this case a new exact solution of the equations of magnetohydrodynamics was obtained: by Lin—on the basis of formal requirements on the structure of the vectors  $\mathbf{v}$ ,  $\mathbf{H}$ ,  $\nabla p'$ , and by Regirer—for the steady flow of a viscous incompressible conducting medium.

In the present work, such magnetic fields are sought for which the following are possible: 1) unsteady flows of a compressible conducting medium; 2) steady flow of a compressible conducting medium; 3) unsteady flows of a viscous incompressible conducting medium; 4) unsteady flow of a viscous compressible conducting medium.

In the third case, for steady flow, Regirer's results are obtained. Analogously to <sup>(3, 4)</sup>, the solutions obtained will be new exact solutions of magnetohydrodynamics.

The equations of magnetohydrodynamics, when the flow of a medium with density  $\rho = \rho(x, y, t)$  occurs along the  $Ox$  axis with velocity  $v_x = u(y, t)$ ,  $v_y = v_z = 0$ , and in the presence of an as yet undetermined plane magnetic field with components  $H_x(x, y, t)$ ,  $H_y(x, y, t)$ , will have the form:

$$\frac{\partial h_x}{\partial x} + \frac{\partial h_y}{\partial y} = 0; \tag{1}$$

$$\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial x} = 0; \tag{2}$$

$$\frac{\partial h_x}{\partial t} + u \frac{\partial h_x}{\partial x} = h_y \frac{\partial u}{\partial y} + \lambda \Delta h_x; \tag{3}$$

$$\frac{\partial h_y}{\partial t} + u \frac{\partial h}{\partial x} = \lambda \Delta h_x; \quad (4)$$

$$\rho \frac{\partial u}{\partial t} = -\frac{\partial p'}{\partial x} + \left( h_x \frac{\partial h_x}{\partial x} + h_y \frac{\partial h_x}{\partial y} \right) + \eta \frac{\partial^2 u}{\partial y^2}; \quad (5)$$

$$\frac{\partial p'}{\partial y} = \left( h_x \frac{\partial h_y}{\partial x} + h_y \frac{\partial h_y}{\partial y} \right), \quad (6)$$

where  $p' = p + h^2/2$  is the total pressure of the medium;  $\lambda = c^2/4\pi\sigma$  is the magnetic viscosity;  $\mathbf{h} = \mathbf{H}/\sqrt{4\pi}$ .

Introducing the vector potential  $A$  and substituting into (1)–(6) the values  $h_x = \partial A/\partial y$ ,  $h_y = -\partial A/\partial x$ , after simple transformations we obtain

$$\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial x} = 0; \quad (7)$$

$$\frac{\partial A}{\partial t} + u \frac{\partial A}{\partial x} = \lambda \Delta A + E(t); \quad (8)$$

$$\frac{\partial}{\partial y} \left( \rho \frac{\partial u}{\partial t} \right) = \frac{D(\Delta A, A)}{D(x, y)} + \eta \frac{\partial^3 u}{\partial y^3}, \quad (9)$$

where  $E(t)$  is proportional to the  $z$ -component of the electric-field vector and, in the general case, is different from zero; in what follows we shall regard it as given;  $D(\Delta A, A)/D(x, y)$  is the Jacobian.

1. If the medium is inviscid, then  $\eta = 0$ , and from (7) we immediately obtain that the density is an arbitrary function  $\rho = F(z)$ , where

$$z = x - \int_0^t u(y, \tau) d\tau.$$

Putting

$$\rho = \alpha z = \alpha \left( x - \int_0^t u(y, \tau) d\tau \right), \quad \alpha = \text{const},$$

from (9), after differentiating with respect to  $x$  and taking into account that  $\partial u/\partial x = 0$ , we shall have

$$\alpha \frac{\partial^2 u}{\partial y \partial t} = \frac{\partial}{\partial x} \frac{D(\Delta A, A)}{D(x, y)}, \quad (10)$$

and since the left-hand side of (10) does not depend on  $x$ , then

$$\frac{\partial^2}{\partial x^2} \frac{D(\Delta A, A)}{D(x, y)} = 0,$$

and from (8)

$$\frac{\partial}{\partial x} \left( \frac{\lambda \Delta A - \partial A / \partial t + E(t)}{\partial A / \partial x} \right) = 0.$$

For the determination of  $A$  two equations have been obtained, which are satisfied by the value

$$A = -x\varphi(y, t) + f(y, t), \quad (11)$$

where  $\varphi$  and  $f$  are determined from a joint system of two equations.

Apparently there are no other solutions for  $A$ , since only (11) ensures the independence of the coefficients of the expressions (10) from  $x$  when the value of  $\Delta A$  from (8) is substituted.

Substituting (11) into (10), we obtain equations for determining  $u, \varphi, f$ :

$$\lambda \alpha \frac{\partial^2 u}{\partial y \partial t} = \frac{\partial^2 \varphi}{\partial y^2} \frac{\partial \varphi}{\partial y} - \varphi \frac{\partial^3 \varphi}{\partial y^3}; \quad (12)$$

$$\frac{\partial \varphi}{\partial t} - \lambda \frac{\partial^2 \varphi}{\partial y^2} = 0; \quad (13)$$

$$\frac{\partial f}{\partial t} - \lambda \frac{\partial^2 f}{\partial y^2} = u\varphi + E(t). \quad (14)$$

Solving the boundary-value problem for the heat-conduction equation, we can find  $\varphi$  from (13), and then  $u$  and  $f$ .  $h$  is determined from

$$h_x = \frac{\partial A}{\partial x} = -x \frac{\partial \varphi}{\partial y} + \frac{\partial f}{\partial y}, \quad h_y = -\frac{\partial A}{\partial x} = \varphi(y, t).$$

2. For the **steady flow of a compressible medium**, the following equations are obtained:

$$u \frac{\partial A}{\partial x} = \lambda \Delta A + E,$$

$$\frac{D(\Delta A, A)}{D(x, y)} = 0.$$

Hence

$$A = x\varphi(y) + f(y),$$

and we obtain the solutions  $\varphi, f, u$  in the form

$$\varphi = ay + b,$$

$$f = \frac{c}{2}y^2 + my + n,$$

$$u = \frac{\lambda c + E}{ay + b},$$

where  $a, b, c, m, n$  are constants.

3. In the case of **nonstationary flow of a viscous conducting fluid** with  $\rho = \text{const}$ , introducing  $\mathbf{h} = \mathbf{H}/\sqrt{4\pi\rho}$ , from (7)–(9) we obtain

$$\frac{\partial A}{\partial t} + u \frac{\partial A}{\partial x} = \lambda \Delta A + E(t); \quad (15)$$

$$\frac{\partial^2 u}{\partial y \partial t} = \frac{D(\Delta A, A)}{D(x, y)} + \nu \frac{\partial^3 u}{\partial y^3}. \quad (16)$$

Taking into account that  $\partial u / \partial x = 0$ , from (15) and (16) we have

$$\frac{\partial}{\partial x} \left[ \frac{\lambda \Delta A - \partial A / \partial t + E(t)}{\partial A / \partial x} \right] = 0,$$

$$\frac{\partial}{\partial x} \frac{D(\Delta A, A)}{D(x, y)} = 0.$$

Hence for  $A$  and  $\varphi$  we obtain the expressions

$$A = -x\varphi(y) + f(y, t),$$

$$\varphi = ay + b,$$

and  $f$  and  $u$  satisfy the equations

$$\lambda \frac{\partial^2 f}{\partial y^2} - \frac{\partial f}{\partial t} = -(ay + b)u - E(t); \quad (17)$$

$$\nu \frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial t} = (ay + b) \frac{\partial^2 f}{\partial y^2} - a \frac{\partial f}{\partial y} - B(t). \quad (18)$$

4. For a **viscous compressible conducting medium**, from (7)–(9) we have

$$A = -x\varphi(y, t) + f(y, t),$$

$$\frac{\partial \varphi}{\partial t} - \lambda \frac{\partial^2 \varphi}{\partial y^2} = 0,$$

$$\frac{\partial f}{\partial t} - \lambda \frac{\partial^2 f}{\partial y^2} = u\varphi + E(t),$$

$$\frac{\partial}{\partial y} \left( \frac{\partial u}{\partial t} - \frac{\eta}{\lambda \alpha} \frac{\partial^2 u}{\partial y^2} \right) = \frac{1}{\lambda \alpha} \left( \frac{\partial^2 \varphi}{\partial y^2} \frac{\partial \varphi}{\partial y} - \varphi \frac{\partial^3 \varphi}{\partial y^3} \right).$$

Developing analogous methods used in papers (5, 6), the solution of nonstationary problems can be reduced to integral equations solvable by successive approximations. In some cases, with the aid of operational calculus, solutions can be obtained in a more compact form.

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