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## Abstract

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Physics

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# ON THE REACTION FORCES CAUSED BY CHERENKOV RADIATION IN A CRYSTAL

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In the present work, Vavilov–Cherenkov radiation is considered for the uniform motion of a charge in an arbitrary direction in a uniaxial nonmagnetic crystal. A similar question was considered in works ([2], [3]); the authors obtained general expressions for the energy losses of a charge due to Vavilov–Cherenkov radiation.* The distinctive feature of the present work is the derivation of expressions for the force causing the deviation of the charge from a rectilinear path, which arises both from the asymmetry of the radiation cone and from the asymmetry of the radiation intensity along its generators. In addition, the formulas given for the braking forces are simpler.

If one uses the gauge of the potentials most convenient for crystals, proposed by V. L. Ginzburg ([1]), $\operatorname{div} \varepsilon \mathbf{A} = 0$, then for a charge $e$ moving in the crystal with velocity $\mathbf{v}$, from Maxwell' s equations we obtain the known expressions for the potentials:

$$\varphi = \frac{e}{2\pi^2 c} \int e^{i\mathbf{k}(\mathbf{r}-\mathbf{v}t)} \frac{d\mathbf{k}}{\mathbf{k}\varepsilon\mathbf{k}},$$

$$\mathbf{A} = -\frac{e}{2\pi^2 c} \int \Lambda^{-1} \left[\mathbf{s} - \mathbf{k}\frac{(\mathbf{k}\varepsilon\Lambda^{-1}\mathbf{s})}{(\mathbf{k}\varepsilon\Lambda^{-1}\mathbf{k})}\right] e^{i\mathbf{k}(\mathbf{r}-\mathbf{v}t)}\, d\mathbf{k}, \tag{1}$$

where $\mathbf{s} = \mathbf{v} - \varepsilon\mathbf{k}(\mathbf{kv})/(\mathbf{k}\varepsilon\mathbf{k})$; $\Lambda = \varepsilon(\mathbf{kv})^2/c^2 - k^2$.

We denote the system of principal axes of the crystal by $XYZ$, and let $\varepsilon_x = \varepsilon = \varepsilon_0$, $\varepsilon_z = \varepsilon_e$. We assume that the charge moves in the plane $YZ$. We introduce a coordinate system $X'Y'Z'$ such that $\mathbf{v}$ is directed along the axis $Z'$, while the axes $X$ and $X'$ coincide. We denote by $\theta$ ($\theta'$) the angle between the vector $\mathbf{k}$ and the axis $Z'$, and by $\varphi$ ($\varphi'$) the angle between the projection of the vector $\mathbf{k}$ onto the plane $XY$ ($X'Y'$) and the axis $X$ ($X'$). Let the angle between the axes $Z$ and $Z'$ be $\alpha$. Then between the trigonometric functions of these angles we have the relation $\cos\theta = \cos\theta' \cos\alpha - \sin\theta' \sin\varphi' \sin\alpha$.

Let us denote $\mathbf{kv} = \omega$, then

$$d\mathbf{k} = \frac{\omega^2}{v^3}\, q\, dq\, d\omega\, d\varphi',$$

where $q = \operatorname{tg}\theta'$. In this case the components of the vector potential $\mathbf{A}$ needed below are equal to

$$A_y = \frac{e}{2\pi^2 c}\int\left\{\frac{\varepsilon_0 \sin\varphi\cos\theta\cos\theta' - M\sin\alpha}{M\left(\varepsilon_0\frac{\omega^2}{c^2} - k^2\right)} - N\frac{\sin\varphi\sin\theta}{\varepsilon_0\frac{\omega^2}{c^2} - k^2}\right\} e^{i\mathbf{k}(\mathbf{r}-\mathbf{v}t)}\frac{\omega^2}{v^2}\, q\, dq\, d\varphi'\, d\omega,$$

$$A_z = \frac{e}{2\pi^2 c}\int\left\{\frac{\varepsilon_e \cos\theta\cos\theta' - M\cos\alpha}{M\left(\varepsilon_e\frac{\omega^2}{c^2} - k^2\right)} - N\frac{\cos\theta}{\varepsilon_e\frac{\omega^2}{c^2} - k^2}\right\} e^{i\mathbf{k}(\mathbf{r}-\mathbf{v}t)}\frac{\omega^2}{v^2}\, q\, dq\, d\varphi'\, d\omega, \tag{2}$$

where

$$M = \varepsilon_0\sin^2\theta + \varepsilon_e\cos^2\theta,$$

$$N = k^2(\varepsilon_e - \varepsilon_0)\cos\theta\sin\theta\cos\varphi'\,\frac{\varepsilon_e\cos\theta\sin\varphi\sin\alpha - \varepsilon_0\sin\theta\cos\alpha}{M\left(\varepsilon_0\varepsilon_e\omega^2/c^2 - k^2 M\right)}.$$

* The formulas for the radiation intensity given in work ([3]) are inaccurate.

The choice of an unambiguous method for integrating these expressions in a transparent medium is made by introducing a small absorption and using, in the limiting transition to a transparent medium, the identity

$$\frac{1}{f(q) + i\varepsilon X} = \frac{P}{f(q)} - \frac{X}{|X|}\, i\pi\delta[f(q)]. \tag{3}$$

Here it is essential that only the second term in (3) is associated with the Vavilov–Cherenkov radiation field.

The force acting on the charge can be obtained by taking the value of the radiation field at the point where the charge is located. With the chosen gauge of the potentials we have:

a) the braking force

$$F_z' = -\frac{e}{c}\left(\frac{\partial A_z}{\partial t}\cos\alpha + \frac{\partial A_y}{\partial t}\sin\alpha\right)_{r\to\mathbf{v}t};$$

b) the deflecting force

$$F'_y = -\frac{e}{c}\left[v\left(\frac{\partial A_y}{\partial z} - \frac{\partial A_z}{\partial y}\right) + \frac{\partial A_y}{\partial t}\cos\alpha - \frac{\partial A_z}{\partial t}\sin\alpha\right]_{r\to\mathbf{v}t}.$$

**Radiation of ordinary waves.** For ordinary waves, only $\partial A_y/\partial t$ and $\partial A_y/\partial z$ are nonzero, since $A_z$ does not contain in the denominator the factor $\varepsilon_0\omega^2/c^2 - k^2$ corresponding to these waves. The radiation cone of ordinary waves has the form $\mathrm{tg}^2\,\theta' = \varepsilon_0\beta^2 - 1$. The requirement that $\mathrm{tg}^2\,\theta' > 0$ is the condition for radiation and ultimately determines the region of integration over $\omega$. The deflecting force acting on the charge in the case of ordinary waves is associated with the asymmetry of the radiation intensity along the generators of the cone, whereas the cone itself is symmetric.

Let us give the expressions for $\partial A_y/\partial t$, $\partial A_y/\partial z$ after integration over $q$:

$$\frac{\partial A_y}{\partial t} = -\frac{e\sin\alpha}{4\pi c}\int \frac{(\varepsilon_0\beta^2-1)\cos^2\varphi'\,|\omega|\,d\omega\,d\varphi'}{\cos^2\alpha - \varepsilon_0\beta^2 + \sin^2\alpha\sin^2\varphi'(\varepsilon_0\beta^2-1) - 2\frac{\omega}{|\omega|}\cos\alpha\sin\alpha\sin\varphi'\sqrt{\varepsilon_0\beta^2-1}},$$

$$\frac{\partial A_y}{\partial z} = -\frac{e\sin^2\alpha}{4\pi c v}\int \frac{(\varepsilon_0\beta^2-1)^{3/2}\cos^2\varphi'\sin\varphi'\,\omega\,d\omega\,d\varphi'}{\cos^2\alpha - \varepsilon_0\beta^2 + \sin^2\alpha\sin^2\varphi'(\varepsilon_0\beta^2-1) - \frac{\omega}{|\omega|}2\cos\alpha\sin\alpha\sin\varphi'\sqrt{\varepsilon_0\beta^2-1}} - \frac{1}{v}\frac{\partial A_y}{\partial t}\cos\alpha. \quad (4)$$

The radiation conditions impose no restrictions on the integration over $\varphi'$, which in these formulas should be carried out from 0 to $2\pi$. As a result of the calculations we obtain the forces of interest to us*, caused by radiation of ordinary waves:

$$F'_z = \begin{cases} -\dfrac{e^2}{c^2}(1-\cos\alpha)\displaystyle\int\omega\,d\omega, & 0 \le \alpha < \mathrm{arc}\,tg\sqrt{\varepsilon_0\beta^2-1}, \\ -\dfrac{e^2}{c^2}\displaystyle\int\left(1-\dfrac{1}{\beta\sqrt{\varepsilon_0}}\right)\omega\,d\omega, & \mathrm{arc}\,tg\sqrt{\varepsilon_0\beta^2-1} < \alpha \le \dfrac{\pi}{2}; \end{cases} \quad (5)$$

$$F'_y = \begin{cases} \dfrac{e^2}{c^2}\dfrac{(1-\cos\alpha)^2}{\sin\alpha}\displaystyle\int\omega\,d\omega, & 0 \le \alpha < \mathrm{arc}\,tg\sqrt{\varepsilon_0\beta^2-1}, \\ \dfrac{e^2\cos\alpha}{c^2\sin\alpha}\displaystyle\int\dfrac{(\beta\sqrt{\varepsilon_0}-1)^2}{\beta\sqrt{\varepsilon_0}}\omega\,d\omega, & \mathrm{arc}\,tg\sqrt{\varepsilon_0\beta^2-1} < \alpha \le \dfrac{\pi}{2}. \end{cases}$$

Here the integration over $\omega$ is carried out over the region: $\omega > 0$, $\varepsilon_0(\omega)\beta^2 > 1$. In addition, a limitation on the region of integration is imposed by the inapplicability of the macroscopic treatment for small wavelengths.

* For $\alpha = 0$ and $\alpha = \pi/2$, formulas (5), as well as (10) and (11), were obtained in [4].

Radiation of extraordinary waves. Extraordinary waves correspond to the zeros of the factor in the denominator of (2) of the integrand of the potential **A**:

$$\varepsilon_0\varepsilon_e\frac{\omega^2}{c^2}-k^2[\varepsilon_0-(\varepsilon_e-\varepsilon_0)\cos^2\theta]. \tag{6}$$

Equating this factor to zero gives the equation for the radiation cone of extraordinary waves

$$C\,\mathrm{tg}^2\,\theta'+2B\,\mathrm{tg}\,\theta'+A+C=0, \tag{7}$$

where

$$A=\varepsilon_0\varepsilon_e\beta^2-(\varepsilon_e-\varepsilon_0)(\cos^2\alpha-\sin^2\alpha\sin^2\varphi'),\quad B=(\varepsilon_e-\varepsilon_0)\sin\alpha\cos\alpha\sin\varphi',$$
$$C=-\varepsilon_0-(\varepsilon_e-\varepsilon_0)\sin^2\alpha\sin^2\varphi'.$$

This equation has two solutions:

$$\text{for } \omega>0 \qquad \mathrm{tg}\,\theta'=\frac{-B-\sqrt{B^2-C^2-AC}}{C};$$

$$\text{for } \omega<0 \qquad \mathrm{tg}\,\theta'=\frac{-B+\sqrt{B^2-C^2-AC}}{C}.$$

The radiation condition is obtained by requiring that $\mathrm{tg}\,\theta'$ be a real quantity, i.e.

$$B^2>C^2+AC \tag{8}$$

or

$$\varepsilon_0\varepsilon_e(\varepsilon_0\beta^2-1)+\varepsilon_0\varepsilon_e\beta^2(\varepsilon_e-\varepsilon_0)\sin^2\alpha\sin^2\varphi'+(\varepsilon_e-\varepsilon_0)\varepsilon_0\sin^2\alpha\cos^2\varphi'>0.$$

In contrast to ordinary waves, this condition imposes restrictions on the region of integration over $\varphi'$.

As noted in work ([2]), for extraordinary waves in a crystal there are interesting features, consisting in the necessity of choosing, in different frequency regions, retarded or advanced potentials as solutions for the field of Vavilov–Cherenkov radiation. The basic criterion for the choice is the sign of the projection, on the radius in the cylindrical coordinate system associated with the moving charge, of the group velocity of the waves $W_r$. In order that the energy flux be directed away from the charge, it is necessary to have $W_r > 0$, which is achieved by choosing the corresponding potentials. It was proved there that the sign of $W_r$ coincides with the sign of

$$\frac{\varepsilon_0 + (\varepsilon_e - \varepsilon_0)(\sin\varphi' \sin\alpha - \operatorname{ctg}\theta' \cos\alpha)\sin\varphi' \sin\alpha}{\varepsilon_0 \varepsilon_e}.$$

With the method of calculation used in the present work, based on identity (3), the sign of the imaginary addition in the factor (6) responsible for the radiation of extraordinary waves changes in accordance with the need to use retarded or advanced potentials. Therefore, in different frequency regions, the sign before the $\delta$-function in (3) changes. The $\delta$-function itself depends on $f(q)$. The integration is performed according to the formula

$$\int \varphi(q)\,\delta[f(q)]\,dq = \sum_s \frac{\varphi(q_s)}{|f'(q_s)|},$$

where $q_s$ are the roots of $f(q) = 0$. Calculation of $f'(q)$ leads to

$$f'(q) = -\{\varepsilon_0 + (\varepsilon_e - \varepsilon_0)(\sin\varphi' \sin\alpha - \operatorname{ctg}\theta' \cos\alpha)\sin\varphi' \sin\alpha\}\varepsilon_0^{-1}.$$

The appearance in the following formulas of the factor $\Omega = \varepsilon_e/|\varepsilon_e|$ is due to all that has been said.

We now give the result of integration with respect to $q$. After some transformations we obtain the formulas of interest to us for the forces acting on the charge and caused by the radiation of extraordinary waves:

$$F_{z'} = -\frac{e^2}{2\pi c^2}\int \Omega \left[1 - \frac{1}{\beta^2\{\varepsilon_0 + (\varepsilon_e - \varepsilon_0)\sin^2\alpha \sin^2\varphi'\}}\right]\omega\,d\omega\,d\varphi' + \frac{e\Omega}{c}\left(\frac{\partial A_y}{\partial t}\right)_{\text{ordinary}} \sin\alpha;$$

$$F_{y'} = -\frac{e^2 \operatorname{ctg}\alpha}{\pi c^2}\int \Omega \left\{1 - \frac{1}{\beta^2[\varepsilon_0 + (\varepsilon_e - \varepsilon_0)\sin^2\alpha \sin^2\varphi']}\right\}\times$$

$$\times \frac{(\varepsilon_e - \varepsilon_0)\sin^2\alpha \sin^2\varphi'}{[\varepsilon_0 + (\varepsilon_e - \varepsilon_0)\sin^2\alpha \sin^2\varphi']}\omega\,d\omega\,d\varphi' + \frac{e}{c}\Omega v \left(\frac{\partial A_y}{\partial z}\right)_{\text{ordinary}} \cos\alpha. \quad (9)$$

The region of integration of these expressions is determined by the radiation condition (8). The terms appearing here, $\left(\frac{\partial A_y}{\partial z}\right)_{\text{ordinary}}$ and $\left(\frac{\partial A_y}{\partial t}\right)_{\text{ordinary}}$, have the form (4), but the integration here is performed with account taken of condition (8). Further computation requires knowledge of the concrete form of the dispersion dependence $\varepsilon_0(\omega)$, $\varepsilon_e(\omega)$.

**Special case.** In the simplest case, when $\varepsilon_0(\omega) > 1$, $\varepsilon_e(\omega) > 1$, and the frequency regions in which the radiation conditions for ordinary and extraordinary waves are fulfilled coincide, in (9) one can carry out the integration over $\varphi'$. As a result we have

$$F_{z'} = -\frac{e^2}{c^2}\int\left\{1 - \frac{1}{\beta^2\sqrt{\varepsilon_0(\varepsilon_0\cos^2\alpha + \varepsilon_e\sin^2\alpha)}}\right\}\omega\, d\omega,$$

$$F_{y'} = \frac{2e^2\operatorname{ctg}\alpha}{c^2}\int\left\{1 - \frac{\varepsilon_0}{\sqrt{\varepsilon_0(\varepsilon_0\cos^2\alpha + \varepsilon_e\sin^2\alpha)}} - \frac{(\varepsilon_e - \varepsilon_0)\sin^2\alpha}{2\beta^2\left(\sqrt{\varepsilon_0(\varepsilon_0\cos^2\alpha + \varepsilon_e\sin^3\alpha)}\right)}\right\}\omega\, d\omega. \quad (10)$$

If $\alpha$ or $\pi/2 - \alpha$ is small, then the expression for $F_{y'}$ can be simplified:

$$\alpha \text{ small.} \qquad F_{y'} = -\frac{e^2}{c^2}\alpha\int\frac{\varepsilon_e - \varepsilon_0}{\varepsilon_0}\left(1 - \frac{1}{\varepsilon_0\beta^2}\right)\omega\, d\omega;$$

$$\pi/2 - \alpha \text{ small:} \quad F_{y'} = -2\frac{e^2}{c^2}\left(\frac{\pi}{2} - \alpha\right)\int\left\{1 - \frac{\varepsilon_0}{\sqrt{\varepsilon_0\varepsilon_e}} - \frac{(\varepsilon_e - \varepsilon_0)}{2\varepsilon_e\beta^2\sqrt{\varepsilon_0\varepsilon_e}}\right\}\omega\, d\omega.$$

From formula (11), in particular, it follows that Čerenkov radiation at small $\alpha$ stabilizes the motion of the particle if $\varepsilon_e > \varepsilon_0$, and destabilizes it if $\varepsilon_e < \varepsilon_0$.

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