CREEP OF ICE AND FROZEN SKELETAL SOILS UNDER A COMPLEX STATE OF STRESS*

1960

SovietRxiv

Abstract

Full Text

CONTINUUM MECHANICS

K. F. VOITKOVSKII

CREEP OF ICE AND FROZEN SKELETAL SOILS UNDER A COMPLEX STATE OF STRESS*

(Presented by Academician D. I. Shcherbakov on 21 III 1960)

In order to study the laws of creep of ice and frozen skeletal soils (sand, gravel), the author carried out a series of long-duration experiments, lasting up to 5000 hr, on compression, tension, bending, pure shear, and complex deformation (by simultaneous torsion and longitudinal compression of tubular specimens) of sandy soils with various ice contents and of pure ice. On the basis of the experiments performed, the basic laws of ice creep were refined ([2]), according to which the rate of steady-state creep of ice under a complex state of stress is characterized by the formula

$$D_{\dot{\varepsilon}} = \frac{KS^{n-1}}{1+|\theta|}\frac{D_\sigma}{2}, \tag{1}$$

where $D_{\dot{\varepsilon}}$ is the deviator of strain rates; D_σ is the deviator of stresses; $S = \{{}^1/{}_6[(\sigma_x-\sigma_y)^2+(\sigma_y-\sigma_z)^2+(\sigma_z-\sigma_x)^2]+\tau_{xy}^2+\tau_{yz}^2+\tau_{zx}^2\}^{1/2}$ is the intensity of shear stresses; θ is the temperature of the ice in °C; n and K are coefficients depending on the structure of the ice (for ice of random structure $n = 1.6$–2.2, $K = (1.6 \div 4.0)\cdot 10^{-5}$ cm^{2n} deg/kg^n hr).

The initial stage of creep, before a constant rate is established, is characterized by the equation (using pure shear as an example)

$$\gamma_t = \gamma^{\text{upr}} + \dot{\gamma}_\infty t\left(1+\frac{at_0}{1+at}\right), \tag{2}$$

where $\dot{\gamma}_\infty$ is the rate of steady-state creep (in accordance with equation (1)); t is time in hours; t_0, a are coefficients ($t_0 = 30 \div 100$ hr, $a \approx 0.5$).

For frozen skeletal soils, until recently it had not been possible to establish clear laws of creep, and a number of questions remained unclear. Thus, it has long been known that in compression frozen sand is usually stronger than ice and frozen loamy soils. At the same time, it turned out that under pure shear and bending the long-term strength limit of sand is less than the strength of frozen loam ([1]), and, with increasing stresses, the creep rate of frozen sand

Fig. 1. Experimental creep curves of ice and frozen sand at −3.5°.

Figure 1: Fig. 1. Experimental creep curves of ice and frozen sand at −3.5°.

reaches the creep rate of pure ice and then may considerably exceed the latter. Under simultaneous shear and compression, the creep rate of frozen sand may differ considerably from the creep rate of pure ice, either downward or upward, depending on the magnitude of the stresses τ and σ and their ratio (Fig. 1).

In the present article, the features of creep of frozen skeletal soils are analyzed, equations for their creep rates are given, and expla—

* Reported on 28 I 1960 at the First All-Union Congress on Theoretical and Applied Mechanics.

···the noted features are explained on the basis of the laws of ice creep. The latter seems possible to us because frozen skeletal soils consist of solid, slightly deformable mineral particles cemented by ice, and their creep is governed by the creep of the ice contained in them. To verify the validity of this approach, experiments were carried out on deformation of skeletal-soil models in the form of metal balls cemented by ice. It turned out that the creep laws of such models correspond to the creep laws of frozen sand (Fig. 2).

Fig. 1. Experimental creep curves of ice and frozen sand at −3.5°. 1 —ice; 2 —frozen sand.
a and *b* —creep under the action of shear (τ) and normal (σ) stresses (torsion and longitudinal compression of tubular specimens); *c* —tension; *d* —bending of beams with cross section 10×10 cm, span 100 cm.

To facilitate the analysis of creep laws, let us divide frozen skeletal soils into two groups depending on the amount of ice in them: 1) ice with inclusions of mineral particles, when the latter are sufficiently separated by ice interlayers and, during deformation, almost do not come into direct contact with one another; 2) soils in which the mineral particles are surrounded by ice but are located close to one another and will come into contact during deformation. Soils with incomplete water saturation, when the particles are not completely cemented and there are many air pores, are not considered here.

We shall first consider the creep laws using the example of **pure shear**.

In accordance with formula (1), for **pure ice** the steady shear rate is determined by the formula

$$\dot{\gamma}_1 = k\tau^n, \tag{3}$$

where k is a generalized coefficient taking into account the structure of the ice and its temperature ($k = K/(1 + |\theta|)$).

Figure 2

Figure 2: Figure 2

In **ice with mineral inclusions**, at small shear stresses the creep laws will basically be the same as for pure ice; only the creep rate may be somewhat lower owing to the relatively smaller amount of ice per unit···

volume. At stresses greater than the long-term strength of freezing of ice to the surface of the mineral particle τ_{cm} (of the order of 0.3 kg/cm^2 at -0.2° and 2.0 kg/cm^2 at -3°), stress concentration will arise in individual ice interlayers, which may lead to an increase in the shear rate as compared with the shear rate of pure ice. Thus,

$$\text{for } \tau \leq \tau_{\text{cm}} \qquad \dot{\gamma}_2 = k_1 \tau^n, \tag{4}$$

where $k_1 \leq k$. For $\tau > \tau_{\text{cm}}$ the calculated shear stress in the ice will be greater than the averaged value:

$$\tau_{\text{ice}} = \tau c_k, \tag{5}$$

where c_k is the calculated coefficient of stress concentration in the ice, which in simplified form may be represented by the equation

$$c_k = 1 + c\frac{\tau - \tau_{\text{cm}}}{\tau} \tag{6}$$

(where c is a coefficient depending on the ratio of the sizes of the mineral particles and the ice interlayers between them, and also on the arrangement and shape of the particles), and the shear rate will be

$$\dot{\gamma}_2 = k c_k^n \tau^n, \tag{7}$$

i.e., greater than that for pure ice by a factor of c_k^n.

Fig. 2. Rate of relative compression of ice, frozen sand, and models of skeletal soils at -3.5° (compression of $65 \times 65 \times 65$ mm cubes).
1 –ice; 2 –metal balls 15 mm in diameter, cemented by ice; 3 –the same, 2-3 mm in diameter; 4 –frozen sand ($\omega = 20\%$).

In a **water-saturated soil**, during creep the mineral particles of the soil come into contact with one another, and then part of the external load (in the present case, the shear force) can be transmitted directly through the mineral particles and thereby reduce the stresses in some ice interlayers between the particles. Individual particles, as it were, interlock with one another, and for further deformation they must move apart; but this is hindered by the cementing action

Figure 3

Figure 3: Figure 3

of the ice. Therefore further deformation is possible only when the stresses exceed a certain limit—the limit of long-term resistance of the soil to shear, τ_{dl}. In this case, further deformation may be associated with the formation of internal defects (ruptures), when the ice interlayers are small in comparison with the mineral particles and do not allow them to come out of interlock without ruptures of continuity.

Fig. 3. Dependence of the steady shear rate, $\dot{\gamma}$, on shear stresses τ.
1 –ice; 2 –ice with inclusions of mineral particles; 3 –skeletal soil: $3a$ $-\sigma_{\text{cr}} = 0$; $3b$ $-\sigma_{\text{cr}} > 0$.

Thus, for $\tau \leq \tau_{\text{dl}}$ decaying creep occurs, in which a redistribution of internal stresses takes place in the soil. The shear stresses in the ice interlayers gradually relax, and the external shear forces are taken up completely by the mineral aggregates; the creep rate then falls to zero.

For $\tau > \tau_{\text{dl}}$, stresses act in the ice interlayers

$$\tau_{\text{ice}} = \tau - \tau_{\text{dl}},$$

and the rate of steady creep will be

$$\dot{\gamma}_3 = k(\tau - \tau_{\text{dl}})^n \quad (8)$$

(the indicated formula was proposed by S. S. Vyalov (3)).

If, however, $\tau - \tau_{\text{dl}} > \tau_{\text{sm}}$, then, analogously to the preceding case, along with the indicated decrease in stresses there will also arise an additional concentration of them in individual ice interlayers, and then

$$\dot{\gamma}_3 = kc_k^n(\tau - \tau_{\text{dl}})^n, \quad (9)$$

where the concentration coefficient c_k will be considerably greater than for ice with mineral inclusions, owing to the decrease in the ratio of the areas of ice interlayers working in shear to the area of sliding over the particle surface. As a result, as the stresses increase, the creep rate of such a soil will reach and exceed the creep rate of pure ice and of ice with mineral inclusions (Fig. 3). As the magnitude of the total deformation increases, the creep rate may also increase owing to the formation of internal defects, as a result of which the stage of steady-state creep will be limited and the deformation will gradually pass into the stage of progressive flow, which was also observed by us in experiments on pure shear of frozen sand ([1]).

Hydrostatic pressure, as the experiments of G. Rigsby ([4]) show, has no substantial effect on the creep rate of ice; therefore the rate of steady-state creep of ice under a **complex stressed state** is determined by the magnitude of the stress deviator D_σ and is practically independent of σ_{sr}. This proposition may also be extended to ice with mineral inclusions; then the steady-state rate of its creep in general form is expressed by the formula

$$D_{\dot{\varepsilon}} = kc_k^n S^{n-1} \frac{D_\sigma}{2}, \tag{10}$$

where $c_k = 1$ for $S \leqslant \tau_{\text{sm}}$ and $c_k > 1$ (according to formula (6)) for $S > \tau_{\text{sm}}$. The initial stage of creep may be described by an equation analogous to equation (2).

As the ice content of the soil decreases, the influence of pressure becomes ever more substantial, since in the process of deformation friction arises between particles, the magnitude of which is proportional to the pressure ($\tau_{\text{tr}} = \sigma_{\text{sr}} \tan\beta$, where $\tan\beta$ is the coefficient of internal friction). Therefore, non-decaying creep in a **frozen skeletal soil** under a complex stressed state can arise only when $S > \tau_{\text{dl}} + \sigma_{\text{sr}} \tan\beta$, and its rate is expressed by the formula

$$D_{\dot{\varepsilon}} = kc_k^n (S - \tau_{\text{dl}} - \sigma_{\text{sr}} \tan\beta)^n \frac{D_\sigma}{2S}, \tag{11}$$

where, for $S - \tau_{\text{dl}} - \sigma_{\text{sr}} \tan\beta \leqslant \tau_{\text{sm}}$, $c_k = 1$, and for $S - \tau_{\text{dl}} - \sigma_{\text{sr}} \tan\beta > \tau_{\text{sm}}$, $c_k > 1$ (the change in volume of the frozen soil due to σ_{sr} is insignificant, and in creep calculations it may be neglected; then $\dot{\varepsilon}_{\text{sr}} = 0$). The last equation describes rather well the above-noted features of the creep of frozen skeletal soils and agrees with experimental data (for frozen sand with a moisture content of about 20% at -3.5°, $\tau_{\text{dl}} \approx 1$ kg/cm^2, $\tan\beta \approx 0.8$, $n = 1.4 \div 2$, $k \approx 10^{-5}$).

Institute of Permafrost Studies named after V. A. Obruchev
Academy of Sciences of the USSR

Received
18 III 1960

CITED LITERATURE

1. K. F. Voitkovskii, Proceedings of the Northeastern Department of the Institute of Permafrost Studies named after V. A. Obruchev, Academy of Sciences of the USSR, issue 1, Yakutsk, 1958.

2. K. F. Voitkovskii, Proceedings of the Siberian Branch of the Academy of Sciences of the USSR, No. 4 (1959).

3. S. S. Vyalov, DAN, 108, No. 6 (1956).

4. G. P. Rigsby, *J. Glaciology*, 3, No. 24 (1958).

Note: Figure translations are in progress. See original paper for figures.