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Abstract

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THE INFLUENCE OF MULTIPLE SCATTERING ON TRANSITION RADIATION

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A charged particle moving through the interface between two media generates so-called transition radiation ([1]), which is usually considered under the assumption of uniform and rectilinear motion. In the relativistic case the energy of transition radiation grows in proportion to the energy of the particle itself ([2,3]). Since the formation of transition quanta occurs over a long segment of the path, multiple scattering can substantially alter both the spectrum and the energy of transition radiation. Attention was drawn to this circumstance in ([4]); however, we consider it necessary to revise the interpretation of the effect given there. Here we shall also estimate the spectrum and the energy of the radiation.

Let us consider the case of the motion of a point charge through an interface between a medium and vacuum. As is known, a relativistic particle radiates energy at small angles, for which the amplitude of the spherical wave of the transition-radiation field is proportional to ([1,5])

$$H \sim \vartheta \left[\left(1 - \beta + \frac{\vartheta^2}{2} \right)^{-1} - \left(1 - n\beta + \frac{\vartheta^2}{2} \right)^{-1} \right] \tag{1}$$

provided that the refractive index of the medium $n = \sqrt{1 - \omega_0^2/\omega^2}$ ($\omega_0^2 = 4\pi e^2 N/m$) is close to unity (the main contribution to the energy is made by high frequencies $\omega > \omega_0$, for which $1 - n \ll 1$, and the dispersion dependence is the same as in a plasma ([2,3])). The first term in (1) corresponds to the field formed along the path in vacuum. The appearance of the second term is due to motion in the medium. The field amplitude is proportional to the difference between the paths of coherent interaction of the particle with the wave in vacuum (s_v) and in the medium (s_c), which depend on the angle ϑ between the direction of wave propagation and the particle velocity and are respectively equal to

$$s_v \sim \frac{c}{\omega} \left(1 - \beta + \frac{\vartheta^2}{2} \right)^{-1}, \quad s_c \sim \frac{c}{\omega} \left(1 - n\beta + \frac{\vartheta^2}{2} \right)^{-1}.$$

In the direction in which the radiation energy density per unit solid angle is maximal $\left(\vartheta \simeq \frac{\mu c^2}{E} \ll 1 \right)$,

$$s_v \sim \frac{c}{\omega}\left(\frac{E}{\mu c^2}\right)^2, \quad s_c \sim \left[\left(\frac{\mu c^2}{E}\right)^2 + \frac{\omega_0^2}{2\omega^2}\right]^{-1}\frac{c}{\omega},$$

where E and μ are the total energy and rest mass of the particle. At frequencies below the critical one, $\omega < \omega_{\text{cr}} = \omega_0 \frac{E}{\mu c^2}$, $s_v \gg s_c$; therefore the transition-radiation field is formed mainly along the path in vacuum. Multiple scattering

in the medium distorts the small term in (1), and therefore it cannot reduce the probability of emission of transition quanta*.

At frequencies $\omega > \omega_{\text{cr}}$, for a non-scattering particle $s_{\text{B}} \sim s_{\text{c}}$, and therefore energy is not radiated. Multiple scattering is essential in the case when, over the path

$$s = \frac{c}{\omega}\left(\frac{E}{\mu c^2}\right)^2$$

the particle is deflected beyond an angle $\vartheta \sim \mu c^2/E$. In this case the path of coherent interaction of the particle with the wave in the medium, s_{c}, will be smaller than without taking scattering into account, and the field corresponding to the second term in formula (1), which causes the cutoff of the spectrum, will not be formed. The radiation spectrum is then enriched by new frequencies**. Thus, the relation must be satisfied

$$\sqrt{\langle\vartheta^2\rangle_{\text{cp}}} > \frac{\mu c^2}{E},$$

where $\langle\vartheta^2\rangle_{\text{cp}}$ is the mean square of the scattering angle over the path s:

$$\langle\vartheta^2\rangle_{\text{cp}} = \left(\frac{E_s}{E}\right)^2 \frac{s}{L}, \qquad E_s = 21 \cdot 10^6 \text{ eV},$$

and L is the radiation unit of length. Rewriting the last inequality in the form

$$\frac{E_{sE}}{(\mu c^2)^2}\sqrt{\frac{c}{\omega L}} > 1. \tag{2}$$

If it is satisfied for the frequency ω_{cr}, i.e.

$$\frac{E_s}{\mu c^2}\sqrt{\frac{E}{\mu c^2}\frac{c}{\omega_0 L}} > 1, \tag{3}$$

then new frequencies appear in the transition-radiation spectrum:

$$\omega_{\text{cr}} < \omega \lesssim \omega_{\text{cr}}^*,$$

for which relation (2) is satisfied, where

$$\omega_{\text{cr}}^* = \frac{E_s^2 E^2}{(\mu c^2)^4} \frac{c}{L}. \tag{4}$$

Let E' denote the particle energy for which $\omega_{\text{cr}}^* = \omega_{\text{cr}}$:

$$E' = \frac{\omega_0 L}{c} \frac{(\mu c^2)^3}{E_s^2}.$$

For $E < E'$ the usual formula for the energy of transition radiation is valid ([2,3]):

$$W = \frac{e^2 \omega_0}{3c} \frac{E}{\mu c^2}.$$

If the character of the spectral cutoff did not change because of scattering, then the energy of transition radiation for $E > E'$ could be obtained from the last formula by replacing $\omega_0 E/\mu c^2$ by ω_{cr}^*. However, the maximum radiated frequency, when scattering is taken into account, falls more rapidly with increasing angle. Indeed, without scattering, in any direction those frequencies are radiated for which the paths of coherent interaction of the particle with the field on the two sides of the interface are substantially different. Let us write the approximate condition for emission ($s_{\text{B}} > 2s_{\text{c}}$):

$$\sqrt{1 + \frac{\vartheta^2}{1-\beta^2}} < \frac{\omega_{\text{cr}}}{\omega}.$$

Because of scattering, the field in the interval of angles smaller than some ϑ, corresponding to the second term in (1), is not formed if, over the path of coherent interaction s_{c} (without taking scattering into account), the particle is deflected beyond this angle. In the region where

$$s_{\text{B}} \sim s_{\text{c}} \sim \frac{2c}{\omega(1-\beta^2+\vartheta^2)},$$

in the direction

$$\vartheta > \sqrt{1-\beta^2}$$

multiple scattering reduces the path of coherent interaction by more than a factor of two

* However, in other directions as well ($\vartheta \neq \mu c^2/E$), transition quanta without scattering are formed in the frequency region in which the second term in formula (1) is small compared with the first.

** We shall also call this radiation transition radiation, since it is associated with the passage of the particle through the interface.

for

$$\left(\frac{E_s}{E}\right)^2 \frac{s_c}{L} > 2\vartheta^2.$$

Taking this relation as an additional radiation condition, valid in order of magnitude for any directions, we rewrite it in the form

$$\frac{\vartheta^2}{1-\beta^2}\left(1+\frac{\vartheta^2}{1-\beta^2}\right) < \frac{\omega^*_{\text{cr}}}{\omega}.$$

Thus, for example, in the region $\vartheta > \sqrt{1-\beta^2}$ the maximum emitted frequency is inversely proportional to the fourth power of ϑ, whereas without taking scattering into account it is inversely proportional to the first power of ϑ.

For estimates we shall use the fact that the field is formed mainly in vacuum and is determined by the first term in (1). The corresponding formula for the radiation energy per unit polar angle and per unit frequency interval has the form

$$\frac{dW_\omega}{d\vartheta} = \frac{2e^2}{\pi c}\frac{\vartheta^3}{(1-\beta^2+\vartheta^2)^2}. \tag{5}$$

The integration must be carried out over the region where at least one of the following inequalities is satisfied:

$$\text{a)}\quad \frac{\vartheta^2}{1-\beta^2}\left(1+\frac{\vartheta^2}{1-\beta^2}\right) < \frac{\omega^*_{\text{cr}}}{\omega}; \qquad \text{b)}\quad \sqrt{1+\frac{\vartheta^2}{1-\beta^2}} < \frac{\omega_{\text{cr}}}{\omega}. \tag{6}$$

Taking the first inequality into account gives a small correction for $E < E'$. Integrating (5) over the region (6b), we obtain: $W \approx 4e^2\omega_{\text{cr}}/3\pi c$, i.e., the known result ([2,3]). For $E \gg E'$ a small correction is made by taking account of inequality (6b). Integrating (5) over the region (6a), we find the spectral density of the radiation energy

$$W_\omega \approx \frac{e^2}{\pi c}\left[\ln\frac{\sqrt{\omega}+\sqrt{\omega+4\omega^*_{\text{cr}}}}{2\sqrt{\omega}} + \frac{2\sqrt{\omega}}{\sqrt{\omega}+\sqrt{\omega+4\omega^*_{\text{cr}}}} - 1\right], \tag{7}$$

the radiation energy density per unit polar angle

$$dW/d\vartheta \approx \frac{2(1-\beta^2)^2 e^2 \omega_{\rm cr}^* \vartheta}{\pi c(1-\beta^2+\vartheta^2)^3},$$

and the total radiation energy

$$W \approx \frac{e^2 \omega_{\rm cr}^*}{2\pi c},$$

where $\omega_{\rm cr}^*$ is determined by formula (4). Thus, the transition-radiation energy for $E > E'$ is, in order of magnitude, equal to

$$W \approx \frac{e^2 E_s^2}{2\pi L(\mu c^2)^2} \left(\frac{E}{\mu c^2}\right)^2. \tag{8}$$

It grows proportionally to the square of the particle energy.

For $E \sim E'$, the cutoff of the spectrum in the angular region essential for integration is due partly to multiple scattering and partly to the polarization of the medium. Because the nature of the cutoff differs, the linear growth of the radiation energy changes to quadratic growth over some finite interval near E', where result (8) is somewhat underestimated. We note that the result of exact integration over the region (6) does not exceed

$$W = \frac{e^2}{\pi c} \left(\frac{4}{3}\omega_{\rm cr} + \frac{1}{2}\omega_{\rm cr}^*\right).$$

Since quanta whose energy is greater than the particle energy are not emitted, formula (8) ceases to be valid when $\hbar\omega_{\rm cr}^* \gtrsim E$ $(E \gtrsim L(\mu c^2)^4/E_s^2\hbar c)$. If $E \gg L(\mu c^2)^4/E_s^2\hbar c$, then the radiation energy according to po-

in order of magnitude is equal to

$$W \approx \int^{E/\hbar} W_\omega \, d\omega \approx \frac{e^2 E}{2\pi\hbar c}\left[\ln\frac{E_s^2 E\hbar c}{L(\mu c^2)^4} - 1\right].$$

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Note: Figure translations are in progress. See original paper for figures.