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Abstract

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THE REAL PART OF THE AMPLITUDE OF ELASTIC SCATTERING AT HIGH ENERGIES

(Presented by Academician N. N. Bogolyubov, 21 IV 1960)

1. In paper ([1]) the "sum rule" was obtained

$$\Delta(\mu) - \Delta(\infty) = \frac{4f^2}{\mu^2} + \frac{1}{2\pi^2}\int_{\mu}^{\infty}\frac{dE}{k}\,[\sigma_-(E) - \sigma_+(E)], \tag{1}$$

where

$$\Delta(E) = [D_-(E) - D_+(E)]/E;$$

$D_\pm(E)$ is the real part of the elastic-scattering amplitude; $\sigma_\pm(E)$ are the cross sections for the interaction of π^+- and π^--mesons with the proton. If the value of the constant $\Delta(\infty)$ is known, relation (1) proves very useful for applications of dispersion relations.

We shall show that $\Delta(\infty) = 0$, and shall consider some consequences of this relation.

Experimental data indicate that the interaction cross sections of particles at high energies become constant or, at any rate, do not increase ([2]). In accordance with these data we shall assume that at energies $E \geq a$ the interaction cross sections have the form

$$\sigma_\pm(E) = \sigma_\pm + \alpha_\pm/E + \beta_\pm/E^2 + f_\pm(E), \tag{2}$$

where $\sigma_\pm$ are the limiting values of the cross sections; $\alpha_\pm$ and $\beta_\pm$ are constant coefficients; $f_\pm(E)$ are small functions tending to zero as E increases.

Within the accuracy of present-day experiments one may put $f_\pm(E) = 0$ at high energies. As will be seen below, the presence of the functions $f_\pm(E)$ does not fundamentally change our arguments.

Taking expression (2) into account, by means of dispersion relations the real part of the elastic-scattering amplitude $D_\pm(E)$ can be represented in the form of a series

$$D(E) = A_1 E \ln E + B_1 E + A_0 \ln E + B_0 + A_{-1} \ln E/E + O(1/E) \quad (3)$$

with known coefficients A and B. Some of these coefficients must be equal to zero. To show this, let us represent the elastic-scattering cross section in the form

$$\sigma_{el} = \pi\lambda^2 \sum_{l=0}^{\infty} (2l+1) \left|1 - \beta_l e^{2i\delta_l}\right|^2 =$$

$$= \pi\lambda^2 \sum_{l=0}^{\infty} (2l+1)(1-\beta_l)^2 + 4\pi\lambda^2 \sum_{l=0}^{\infty} (2l+1)\beta_l \sin^2\delta_l = \Sigma_1 + \Sigma_2. \quad (4)$$

Here the first term is entirely due to inelastic processes ($\beta_l \neq 1$) and represents the cross section of purely diffractive scattering. The second term is associated with nondiffractive scattering.

For $l \gg 1$,

$$\Sigma_2 \simeq 8\pi\lambda^2 \int_0^{\infty} \beta_l \delta_l^2 \, dl \simeq 8\pi \int_0^R \rho\beta(\rho E)\delta^2(\rho E) \, d\rho = \beta(E)\delta^2(E) \, \mathrm{const}, \quad (5)$$

where $\rho = \lambda l = l/E$ is the impact parameter, and $\beta(E)$ and $\delta(E)$ are the mean values of the functions β_l and δ_l in the region of action of the nuclear forces. (Clearly, $\beta_l \simeq 0$ for $l > R/\lambda$, where R is the effective radius of the nuclear forces.)

Since at high energies all scattering becomes purely diffractive ([3]), $\beta(E) \to$ const, $\delta(E) \to 0$. It follows from this that the real part of the amplitude

$$D(E) = 2\lambda \sum_{l=0}^{\infty} (2l+1)\beta_l \sin 2\delta_l \simeq E\delta(E) \cdot \mathrm{const} \quad (6)$$

in any case grows more slowly than E, and the coefficients A_1 and B_1 in expansion (3) must be equal to zero:

$$A_1 = \frac{1}{4\pi^2}(\sigma_+ - \sigma_-) = 0; \quad (7)$$

$$B_1 \equiv \frac{1}{2\mu}(D_-^0 - D_+^0) - \frac{2f^2}{\mu^2} + \frac{1}{4\pi^2} \int_\mu \frac{dE}{k} \left[\sigma_+(E) - \sigma_-(E)\right] +$$

$$+\frac{1}{4\pi^2 a}\left[\alpha_+ - \alpha_- + \frac{1}{2a}(\beta_+ - \beta_-)\right] = 0. \quad (8)$$

The first of these relations was previously obtained by I. Ya. Pomeranchuk ([4])*, and from comparison of the second with (1) for $a \to \infty$ it follows that $\Delta(\infty) = 0$.

2. At present there is no theory that would determine how rapidly the non-diffractive part of the scattering Σ_2 decreases with increasing energy. The optical model often used for interpreting experimental data gives one or another energy behavior of this cross section depending on various assumptions about the structure of the interacting particles. However, the inverse formulation of the problem is possible: on the basis of experimental data, to obtain information about the dependence $\Sigma_2(E)$.

Analysis of the data presently known on the interaction of π-mesons with nucleons leads to the conclusion that the cross section Σ_2 decreases with increasing energy, in any case, no faster than $1/E^2$. Indeed, by virtue of relations (5) and (6), a faster decrease of Σ_2 is possible only under the condition that the coefficients A_0 and B_0 in expansion (3) vanish:

$$A_0 \equiv -\frac{1}{4\pi^2}(\alpha_+ + \alpha_-) = 0; \quad (9)$$

$$B_0 = \frac{1}{2}(D_-^0 + D_+^0) + \frac{f^2}{M} - \frac{1}{4\pi^2}\int_\mu^a \frac{E\,dE}{k}\left[\sigma_+(E) + \sigma_-(E)\right] +$$

$$+\frac{a}{4\pi^2}\left[2\sigma - \frac{1}{a^2}(\beta_+ + \beta_-)\right] = 0, \quad (10)$$

where $\sigma_+ = \sigma_- = \sigma$.

* This conclusion also follows from the work of S. Z. Belen' kii ([6]), where it is shown that for $E \to \infty$ the real part of the elastic-scattering amplitude is much smaller than its imaginary part: $D(E) \ll A(E) \simeq \sigma_t E/4\pi$. However, from this it still cannot be concluded that $D(E)$ grows more slowly than E.

We give the values of B_0, calculated from the experimental values of the cross sections $\sigma_\pm(E)$ (see (2)), for $D_-^0 = (0.121 \pm 0.018) \cdot 10^{-13}$ cm; $D_+^0 = -(0.170 \pm 0.016) \cdot 10^{-13}$ cm; $f^2 = 0.08 \pm 0.01$*.

a, BeV	2	3	4	5	> 5
B_0, 10^{-13} cm	−0.37	−0.36	−0.34	−0.33	−0.33

As can be seen, these values differ noticeably from zero. One can obtain the values $B_0 = 0$ if one assumes that in the experimental values of the cross sections $\sigma_\pm(E)$ at energies $E < a$ there is a systematic error which overestimates the values of these cross sections by 10%; or if one assumes that equality of the cross sections $\sigma_+ = \sigma_-$ sets in only at $E > a \gg (3 \div 4)$ BeV, while in the interval 4 BeV $< E < a$ the cross-section values decrease noticeably. Both of these possibilities appear unlikely: the experimental errors $\Delta\sigma_\pm$ indicated by the authors for $E < (4 \div 5)$ BeV amount on average to only 5% and statistically have different signs at different values of the energy E, while measurements of the cross sections of the $(\pi^- p)$ interaction at energies $E = (4 \div 7)$ BeV do not show any appreciable change in them ([2]).

Within the limits of experimental errors, good agreement with experiment can be obtained if one assumes that at high energies

$$\Sigma_2 = \text{const}/E^2, \qquad D_\pm = \text{const} \simeq -0.3 \cdot 10^{-13} \text{ cm.}$$

Correspondingly, in the center-of-mass system of the colliding π-meson and nucleon,

$$D^c_\pm(E) = D_\pm \frac{\lambda_0}{\lambda_c} \frac{\mu}{E} \simeq D_\pm \frac{\mu}{\sqrt{E}} \frac{1}{\lambda_N} \simeq -\frac{0.22}{\sqrt{E}} \cdot 10^{-13} \text{ cm,}$$

where $\lambda_0 = \hbar/\mu c$; $\lambda_N = \hbar/\mu c = 0.21 \cdot 10^{-13}$ cm; μ is the mass of the π-meson; λ_c is its wavelength in the center-of-mass system; E is the π-meson energy in the laboratory coordinate system in BeV.

It is of great interest to measure the magnitude of the elastic-interaction cross section of π-mesons with nucleons at very small scattering angles $\theta \to 0$. We note that the method developed by P. K. Markov et al. ([5]) makes it possible to perform such measurements.

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* The error in the values of B_0, corresponding to the errors $\Delta D_{\pm}^{0}$ and Δf^2, is $\Delta B_0 = \pm 0.02 \cdot 10^{-13}$ cm.

Note: Figure translations are in progress. See original paper for figures.