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Abstract

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GEOPHYSICS

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FURTHER ON EDDY CURRENTS IN THE SEA

As we stated in paper ([1]), the increase in the density of horizontal telluric currents in the ocean ([2]) with descent into the depths compels one to expect some kinship between them and eddy currents, well known in technology.

Up to the present time it has not been possible in any way to localize the hypothetical dynamo that creates the effect ([2]), although the effect itself has been observed in two oceans—the Atlantic and the Indian—on three vessels, in four expeditions, by four authors, and was recorded on an electronic-tube potentiometer ([1]). Therefore it seems useful to us, for the time being, to consider a related phenomenon that at present has been sufficiently well studied instrumentally, namely: the mechanism of the origin of **alternating** electric currents in a closed sea, where constant components of planetary scale cannot arise.

In order to obtain a solution in quadratures, we shall regard the form of the sea as circular and its depths as very small; consequently, the current density varies only as a function of the radius vector and does not depend either on the azimuth or on the depth of the point. For the same reason we shall take into account the oscillations of only one vertical component Z of the earth' s magnetic field in the region of the sea, arising during magnetic storms. When Z increases, ring currents should arise in the sea, left-handed with respect to the direction of the magnetic flux, and when it decreases—right-handed.

We shall consider the inhomogeneities of the magnetic field itself within the limits of the sea area to be so small that everywhere one may assume

$$\operatorname{rot} Z = 0. \tag{1}$$

Although the electrical conductivity σ of water is very small, nevertheless at the frequencies of oscillation under investigation the displacement currents are negligibly small in comparison with the conduction currents. On the other hand, the magnetic permeability of seawater should be set equal to unity. Thus, Maxwell' s equations for an aqueous medium are written in the simple form:

$$\frac{4\pi\sigma}{c}\mathbf{E} = \operatorname{rot} \mathbf{H}, \tag{2}$$

$$\frac{1}{c}\frac{\partial \mathbf{H}}{\partial t} = -\operatorname{rot}\mathbf{E}. \tag{3}$$

Let us form the expression for the curl of both parts of equation (3) and change the order of operations in the left-hand side:

$$\frac{1}{c}\frac{\partial}{\partial t}(\operatorname{rot}\mathbf{H}) = -\operatorname{rot}\operatorname{rot}\mathbf{E}. \tag{4}$$

But on the basis of (2) the left-hand side of the new equation (4) is equal to $\frac{4\pi\sigma}{c^2}\frac{\partial \mathbf{E}}{\partial t}$, while the right-hand side is equal to $\nabla^2\mathbf{E} - \operatorname{grad}\operatorname{div}\mathbf{E}$. On the other hand, in the absence of volume charges in the aqueous medium the divergence of the vector $\mathbf{E}$ must be equal to zero. Consequently, instead of (4) one must write:

$$\frac{4\pi\sigma}{c^2}\frac{\partial \mathbf{E}}{\partial t} = \nabla^2\mathbf{E}. \tag{5}$$

Let us note that, on the basis of (1), $\operatorname{rot}\mathbf{H}$ in the original equation (4) depends only on the structure of the magnetic field of the currents induced in seawater.

Let the external—geomagnetic—field during a magnetic storm oscillate according to a sinusoidal law with cyclic frequency ω. Solving our planar-

problem, let us place the pole of the coordinate system at the center of the sea. Then the instantaneous value of the electric-field strength vector $\mathbf{E}$ at any point of the sea will obviously be represented as the product of the amplitude of the oscillations—a function only of the radius vector $\mathbf{E}_r$—by a complex function of time t:

$$\mathbf{E} = \mathbf{E}_r e^{i\omega t}. \tag{6}$$

Under this condition the left-hand side of (5) gives

$$\frac{4\pi\sigma}{c^2}\frac{\partial \mathbf{E}}{\partial t} = 2ik^2\mathbf{E}_r, \tag{7}$$

where

$$k = \sqrt{2\pi\sigma\omega}, \tag{8}$$

if we agree to measure the electrical conductivity in electrostatic-system units, so as not to introduce the square of the speed of light into the formulas.

We decompose the vector $\nabla^2 \mathbf{E}_r$ on the right-hand side of (5) into components along the axes of a rectangular coordinate system with origin at the pole. Along the same axes we decompose the vector $\mathbf{E}_r$ into components: $X = -E_r \cos\psi$, $Y = E_r \sin\psi$. We apply the usual transformation formulas to the scalar quantities $(\nabla^2 E)_x$ and $(\nabla^2 E)_y$. Then we obtain:

$$(\nabla^2 E)_x = \frac{\partial^2 X}{\partial r^2} + \frac{1}{r}\frac{\partial X}{\partial r} + \frac{1}{r^2}\frac{\partial^2 X}{\partial \psi^2} = -\left(\frac{\partial^2 E_r}{\partial r^2} + \frac{1}{r}\frac{\partial E_r}{\partial r} - \frac{1}{r^2}E_r\right)\cos\psi. \quad (9)$$

Similarly,

$$(\nabla^2 E)_y = \left(\frac{\partial^2 E_r}{\partial r^2} + \frac{1}{r}\frac{\partial E_r}{\partial r} - \frac{1}{r^2}E_r\right)\sin\psi. \quad (10)$$

The required vector is expressed through its projections. Its modulus will be:

$$\nabla^2 E = \sqrt{(\nabla^2 E)_x^2 + (\nabla^2 E)_y^2}. \quad (11)$$

The directions of the vectors $\nabla^2 \mathbf{E}$ and $\mathbf{E}$ coincide with one another. Hence, on the basis of (7), (9), (10), and (11), instead of (5) one writes:

$$\frac{\partial^2 E_r}{\partial r^2} + \frac{1}{r}\frac{\partial E_r}{\partial r} - \left(\frac{1}{r^2} + 2ik^2\right)E_r = 0. \quad (12)$$

For values of the parameter k that occur under natural conditions (as we shall see below), one may assume that, at distances not too small from the center of the sea, $\frac{1}{r^2} \ll 2k^2$. In that case, with sufficient approximation the integral of equation (12) is expressed in terms of a Bessel function of a complex variable:

$$E_r = AJ_0[kr(1-i)] = AJ_0\left(\sqrt{2}kre^{-i\frac{\pi}{4}}\right). \quad (13)$$

The values of this function are tabulated in the handbook [(3)] for various values of the multiplier before i in the exponent of e, from 0 to $\pi/2$ in intervals of 5°, and for values of the argument from 0 to 10. The tables give the numerical values of the real and imaginary parts, which we shall denote respectively by a and b. In (13) we set $A = 1$.

In view of the negative sign before i in (13), in our problem we shall have

$$E_r = a - ib.$$

Fig. 1

Figure 1: Fig. 1

On the basis of (6) it remains to multiply this complex quantity by $e^{i\omega t}$. Then, after discarding the imaginary part, the instantaneous value of E is finally expressed as

$$E = (a - ib)e^{i\omega t} = a\cos\omega t + b\sin\omega t = \sqrt{a^2 + b^2}\cos(\omega t - \delta), \qquad (14)$$

where the phase shift δ, in turn, will be

$$\delta = \operatorname{arc\,tg}\frac{b}{a}. \qquad (15)$$

In Fig. 1, curve 1 represents the increase in the amplitude of oscillations of the electric-field intensity in the sea as the argument $\sqrt{2kr}$ increases, i.e., with distance from the center of the sea. Curve 2 shows how the phase shift δ increases in this process. In the same diagram, curve 3 records the electric field at different distances from the center of the sea at the instant $t = 0$. The curves are interrupted at the value of the argument $\sqrt{2kr} = 2$, in accordance with the remark concerning equation (12). At small distances from the center of the sea one must assume $2k^2 \ll 1/r^2$, in view of which the integral of equation (12) is expressed, with sufficient approximation, in the general form

$$E_r = C_1 r + \frac{C_2}{r}. \qquad (16)$$

It is quite obvious that the arbitrary constant $C_2 = 0$, and therefore, as one approaches the very center of the sea, one should expect the electric-field intensity to fall to zero according to a simple linear law—as is shown schematically by the dashed line in Fig. 1.

Fig. 1

It is of interest to compare the exact integral (12), represented by the curves in Fig. 1, with the approximate solution given by the theory of Bessel functions for the coastal strip of the sea. Let us denote the radius of the shoreline by R, and the distance from the shore to the ring zone under investigation by x. If $x \ll R$, then the approximate integral of (12) is written in the form

$$\frac{E}{E_0} = \sqrt{\frac{R}{R - x}}\, e^{-kx}\cos(\omega t + kx). \qquad (17)$$

Here E_0 is the amplitude of oscillations of the electric-field intensity in the water at the very shore, and the time t is counted from one of the instants when the

Fig. 2

Figure 2: Fig. 2

field intensity at the shore passes through its amplitude value. In practical calculations of the field in the most interesting, coastal, zone one may take $\sqrt{R/(R-x)} \simeq 1$. This assumption was made in computing the curves in Fig. 2 by formula (17). Here curve 1 shows how sharply the electric-field intensity falls off with distance from the shore. The ordinates are the dimensionless quantities E_r/E_0. Along the abscissa are plotted dimensionless distances from the shore—the quantities kx.

We shall express the electrical conductivity of seawater σ in reciprocal ohm-centimeters. Then, instead of (8), one must write the expression

$$k = \sqrt{2\pi\, 10^{-9} \sigma\omega} = 2\pi \sqrt{\frac{\sigma}{T}\, 10^{-9}}, \tag{8'}$$

in which T denotes the period of oscillations of the vertical component of the Earth' s magnetic field in the sea region.

During magnetic storms, complex oscillations with different periods are observed. Correspondingly, already in the first instrumental records of telluric currents in the sea ([4]), oscillations of the intensity are visible

of the electric field in seawater with periods of the order of 3000, 300, and 10 sec. Applying this to the first value, with seawater conductivity 0.03 reciprocal ohm-centimeters, we obtain, for $kx = 1$, a distance of about 55 km from the shore; applying it to the second value of the oscillation period, kx will be equal to 1 at a distance of 17.3 km from the shore; finally, for a period of 10 sec, kx is equal to 1 at a distance of only 1.1 km from the shore. It is precisely at such distances, respectively, that the electric-field intensity in the water must decrease by a factor of e in comparison with its amplitude value at the shore itself.

Fig. 2

In the same Fig. 2, curve 2 represents the distribution of the electric field at various distances from the shore at the moment when the field intensity at the shore passes through a maximum. As we see, curve 1 in Fig. 2 is entirely analogous to curve 1 in Fig. 1, and curve 2 in Fig. 2 is entirely analogous to curve 3 in Fig. 1. Curve 2 in Fig. 1 indicates a linear law of variation of the phase shift δ as the argument changes over wide limits, quite analogous to the change of the phase shift in the approximate formula (17).

In Fig. 1 it is seen that the phase difference, measured on the ordinate axis with respect to the two points of intersection of curve 3 with the abscissa axis, is equal to π, in complete agreement with the cosine law of the same formula.

The difference of the abscissas of these points, divided by $\sqrt{2}$, is also equal to π. Consequently, the exact integral (12) indicates the existence of a peculiar **wave** with increasing amplitude, which propagates from the open sea toward the shore with a velocity readily determined on the basis of (17). Namely, the velocity v with which oscillations of the electric-field intensity in the water advance toward the shore will be

Wave length

$$v = \frac{\omega}{k} = \sqrt{\frac{10^9}{\sigma T}} \text{ cm/sec} = \frac{316}{\sqrt{\sigma T}} \text{ m/sec.} \tag{18}$$

$$\lambda = 0.316\sqrt{\frac{T}{\sigma}} \text{ km.} \tag{19}$$

For $\sigma = 0.03$ and an oscillation period of 3000 sec, (18) gives $v = 33.3$ m/sec, and from (19) we obtain $\lambda = 100$ km; for $T = 300$ sec, $v = 105$ m/sec and $\lambda = 31.6$ km; for $T = 10$ sec, $v = 576$ m/sec and $\lambda = 5.8$ km.

Thus, for the existing periods of oscillation of the vertical component of the terrestrial magnetic field, eddy currents directed along the coastline must be induced in an enclosed sea, with a sharply expressed skin effect, which here may be called a coastal effect.

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4. See the curves of A. T. Mironov in the book: V. V. Shuleikin, *A Short Course in the Physics of the Sea*, L., 1959, p. 459.

Note: Figure translations are in progress. See original paper for figures.