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ANISOTROPY OF THE GALVANOMAGNETIC EFFECT IN AN n-TYPE GERMANIUM CRYSTAL AT TEMPERATURES OF THE TRANSITION REGION OF CONDUCTIVITY

In our previous works ([1], [2]), the longitudinal and transverse galvanomagnetic effects were investigated on one and the same specimen of an n-type germanium single crystal along the principal crystallographic axes at room temperature. The presence of anisotropy with respect to the galvanomagnetic effect in the object under investigation was experimentally confirmed. The aim of the present work is to investigate the anisotropy of this effect in an n-type germanium single crystal at temperatures in the region transitional from impurity conductivity to intrinsic conductivity, which has not been studied up to now.

The theory of the galvanomagnetic effect, allowing for anisotropy in the form of isoenergetic surfaces, has been developed mainly for electron and hole semiconductors of the cubic system ([3–6]), but has not yet been developed as applied to semiconductors of the germanium and silicon type with respect to the transition and intrinsic regions of conductivity. Therefore, we can compare with theory only the experimental data obtained in the region of impurity conductivity. For this purpose we derived ([2]) the formula for the galvanomagnetic effect in the form

$$\frac{\Delta\rho}{\rho_H} = \left[k_0 \sum_{i=1}^{3} g_i^2 h_i^2 + k_1 \sum_{i+j=1}^{3} \left(g_i^2 h_j^2 - g_i g_j h_i h_j \right) \right] H^2, \tag{1}$$

where g_i and h_i are the direction cosines of the unit vectors of the electric current and magnetic field, respectively, with respect to the tetragonal axes of the crystal of the cubic system; k_0 and k_1 are constants that are functions of the ratio of the longitudinal effective electron mass to the transverse one, $m_\parallel^*/m_\perp^* = r$, i.e.

$$k_0 = \frac{8}{3\pi}\frac{u^2}{c^2}\frac{(2r+1)(r-1)^2}{r(r+2)^2}, \qquad k_1 = \frac{4}{3\pi}\frac{u^2}{c^2}\frac{(2r+1)(2r^2+5r+2)}{r(r+2)^2}, \tag{2}$$

u is the mobility of the current carriers, and c is the speed of light.

As is seen from formula (1), the magnitude of the effect is proportional to the square of the magnetic-field strength, i.e. $\Delta\rho/\rho_H = bH^2$, and is different for different crystallographic directions.

Calculations by formula (1) for a specified value of the field give the following ratio of the coefficients b along the axes [100], [110], and [111]:

$$b_{[001]\parallel} = 2b_{[110]\parallel} = 3b_{[111]\parallel}. \tag{3}$$

For transverse effects with respect to the indicated axes in the plane $(\bar{1}10)$, from the theoretical formula (1) we obtain

$$b_{[001]\perp} = b_{[110]\perp} < b_{[111]\perp}. \tag{4}$$

The theoretical coefficient b contains the parameter r and is proportional to the square of the mobility. The simple theory of semiconductors ([7], [8]), taking into account the scattering of thermal vibrations of the lattice, gives for the temperature dependence of the mobility $u = aT^{-3/2}$, where a is a constant. Sub-

substituting the value of u into formulas (2) and introducing the new designations

$$k_0 = k_0' T^{-3}, \qquad k_1 = k_1' T^{-3}, \tag{5}$$

we obtain from (1) for the coefficient of the galvanomagnetic effect

$$\left[\frac{\Delta\rho}{\rho_H H^2}\right] = b = \left[k_0' \sum_{i=1}^{3} g_i^2 h_i^2 + k_1' \sum_{\substack{i,j=1 \\ i\neq j}}^{3} \left(g_i^2 h_j^2 - g_i g_j h_i h_j\right)\right] T^{-3}. \tag{6}$$

Thus, in the region of impurity conduction in weak fields the magnitude of the effect is inversely proportional to the absolute temperature to the third power, and the dependence $b = f(T^{-3})$, both for the longitudinal and for the transverse effect, is different for different crystallographic directions. The germanium sample under investigation was taken in the form of a sphere 29.5 mm in diameter, cut from a large homogeneous single crystal with a specific resistance of about 18 $\Omega \cdot$ cm (donor-electron concentration about 10^{14} cm^{-3}). The data obtained make it possible to conclude that at room temperature and below, the principal current carriers in the specimen are impurity electrons, while above room temperature there is a transition to the region of intrinsic conduction. This conclusion was checked by measuring the temperature dependence of the electrical

Fig. 1

Figure 1: Fig. 1

resistance of the crystal. The calculated activation energy and band-gap width proved to be, respectively, $\Delta E_n \simeq 0.04$ eV and $\Delta E_0 \simeq 0.75$ eV.

Fig. 1. Dependence of $\Delta R/R_H$ on H^2 for the longitudinal galvanomagnetic effect along the principal crystallographic axes of a germanium crystal. 1—0°; 2—20°; 3—50.5°; 4—95°.

The results of the investigation of the longitudinal and transverse galvanomagnetic effects at different temperatures along the principal crystallographic axes are given in Figs. 1 and 2. We note that at all temperatures we found anisotropy in the germanium crystal with respect to the longitudinal and transverse galvanomagnetic effects. The differences in the magnitudes of the effects along the principal crystallographic axes (with the exception of the results corresponding to the temperature 95°) are in agreement with the earlier conclusions of the authors ([1]) for germanium. It follows from formula (1) that the magnitudes of the transverse effect with respect to the axes [001] and [110] at a given magnetic field should be equal; however, the experimental values differ.

In weak fields the magnitudes of the effects depend linearly on H^2. From the tangent of the angle of inclination of the straight lines, the proportionality coefficient $b = \Delta R/R_H H^2$ was calculated. The numerical results are given in Table 1.

The experimental ratios of the longitudinal galvanomagnetic effect differed from those obtained by the theoretical formula (3): the magnitudes of the effect along the three principal crystallographic axes [001], [110], and [111] are larger than the corresponding theoretical ones by the amount β, $b_{[001]\|} = 2b_{[110]\|} - \beta = 3b_{[111]\|} - 2\beta$.

Table 1

Current direction	Magnetic-field direction	b_{theor}	$b_{\text{exp}} \cdot 10^9$, oerst^{-2}	$b_{\text{exp}} \cdot 10^9$, oerst^{-2}	$b_{\text{exp}} \cdot 10^9$, oerst^{-2}	$b_{\text{exp}} \cdot 10^9$, oerst^{-2}
Current direction	Magnetic-field direction	b_{theor}	273°K	293°K	323.5°K	368°K
[001]	[001]	k_0	2.55	2.27	1.79	0.85
[001]	[110]	k_1	1.37	1.34	1.21	1.01
[110]	[110]	$^1/_2 k_0$	1.97	1.86	1.46	0.95
[110]	[001]	k_1	1.48	1.42	1.24	0.82

Figure 2: Dependence of $\Delta R/R_H$ on H^2 for the transverse galvanomagnetic effect along the principal crystallographic axes. The designations are the same as in Fig. 1.

Figure 2: Figure 2: Dependence of $\Delta R/R_H$ on H^2 for the transverse galvanomagnetic effect along the principal crystallographic axes. The designations are the same as in Fig. 1.

Current direction	Magnetic-field direction	b_{theor}	$b_{\text{exp}} \cdot 10^9$, oerst^{-2}	$b_{\text{exp}} \cdot 10^9$, oerst^{-2}	$b_{\text{exp}} \cdot 10^9$, oerst^{-2}	$b_{\text{exp}} \cdot 10^9$, oerst^{-2}
[111]	[111]	$^1/_3 k_0$	1.85	1.73	1.43	0.75
[111]	[11$\bar{2}$]	$k_1 + {}^1/_3 k_0$	1.75	1.71	1.53	0.85

In the impurity-conduction region these ratios quite well satisfy formula (3) for $\beta = 1.44 \cdot 10^{-9}$.

In the impurity-conduction region, the results of experimental studies of the transverse galvanomagnetic effect agree with the theory that assumes anisotropy in the form of the isoenergy surfaces of electrons in the conduction band of germanium. Comparing the ratios of the transverse effects relative to the axes [001] and [111] with theory and eliminating the mobility values, we were able to calculate the ratio of the longitudinal effective mass of the electron to the transverse one and obtained $r = 22$, which agrees well with cyclotron-resonance data ([9]).

With increasing temperature above room temperature, a decrease in r is observed. Thus, for example, from the data corresponding to a temperature of 50.5°C, $r = 19$ was obtained. This is apparently connected with the fact that holes begin to participate in the conductivity; their isoenergy surfaces in the valence band have a more complex form, and, consequently, in our calculations there resulted, as it were, a shortening of the major axis of the ellipsoid of revolution with a corresponding lengthening of the minor axes. Similar calculations at high temperatures lose their meaning, since with the participation in the conductivity of holes of considerable concentration the physical picture of the anisotropy becomes still more complicated.

Fig. 2. Dependence of $\Delta R/R_H$ on H^2 for the transverse galvanomagnetic effect along the principal crystallographic axes. The designations are the same as in Fig. 1.

According to theory, r can be determined in several ways; however, the results may differ. Recently Della-Pergola and Sette ([10]) indicated for r values ranging from 8 to 12. It appears that r, calculat-

Fig. 3

Figure 3: Fig. 3

Fig. 4

Figure 4: Fig. 4

obtained from measurements of the Hall effect and of the transverse galvanomagnetic effect with respect to the given axis may have a smaller value than that obtained by the methods just discussed, since in some cases the electron mobility in germanium as determined from the galvanomagnetic effect proves to be smaller than from the Hall effect.

Fig. 3. Dependence of the coefficients $b = \Delta R/R_H H^2$ on T^{-3} along the principal crystallographic axes of a germanium crystal. I—for the longitudinal effect, II—for the transverse effect.

In Fig. 3 the curves $b = \Delta R/R_H H^2 = f(T^{-3})$ are presented for the longitudinal and transverse galvanomagnetic effects along the principal crystallographic axes of a germanium crystal. As was to be expected, in accordance with formula (6), in both cases different curves are obtained for different crystallographic directions. At the temperature of the transition region a sharp bend is observed in the course of the curves $b = f(T^{-3})$ (especially in the case of the transverse effect).

The obtained values $r = m^*_\parallel/m^*_\perp$ make it possible to calculate the carrier mobility from the galvanomagnetic effect, u_g. As shown in work (2), the mobility value calculated in this way at room temperature proved to be approximately 15-20% smaller than the values obtained by other authors. Within this discrepancy we succeeded in constructing the temperature dependence of the mobility, shown in Fig. 4. Since the theoretical value of the galvanomagnetic effect is proportional to the square of the mobility of the current carriers, and at temperatures above room temperature holes with a concentration comparable with the electron concentration participate in the conductivity, in this region we are dealing with a certain resultant mobility. Nevertheless, in this region the mobility determined from the galvanomagnetic effect changes with temperature according to a dependence close to the theoretical one (the dashed line in Fig. 4), i.e. $u_g \sim T^{-1.49}$. With decreasing temperature the course of the curve changes in the direction of a decrease in u_g. This is possibly connected with scattering by ionized impurities, although in the temperature range considered scattering by thermal vibrations of the lattice is predominant.

Fig. 4. Temperature dependence of the mobility according to the galvanomagnetic effect.

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Note: Figure translations are in progress. See original paper for figures.