ON THE QUESTION OF THE WAVE RESISTANCE OF A BODY IN ITS CIRCULAR MOTION

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Abstract

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ON THE QUESTION OF THE WAVE RESISTANCE OF A BODY IN ITS CIRCULAR MOTION

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Let an arbitrary rigid body move horizontally along a circular path with constant angular velocity ω beneath the free surface of an ideal heavy liquid of infinite depth. Cases are considered of the motion of the body in a circular basin (I), in a circular channel (II), and outside a circular cylinder (III). The last case also includes the motion of a body in an unbounded liquid with a free surface (III$'$). The problem is solved in the linear formulation. In addition, it is assumed that the motion of the body has been going on for an infinitely long time, so that in axes attached to the body the velocity potential of the liquid is stationary. The plane oxy is taken as the undisturbed surface of the liquid, and the axis oz is directed downward along the axis of the body' s trajectory. We assume that the velocity potential φ_1 for the motion of the body in an unbounded liquid is known. Usually it is given in the form

$$\varphi_1(x,y,z) = \iint_S \frac{\gamma(x_1,y_1,z_1)}{R}\,dS; \qquad R^2 = (x-x_1)^2 + (y-y_1)^2 + (z-z_1)^2.$$

The required velocity potential of the wave motion will be

$$\varphi(r,\vartheta,z) = \begin{cases} -8\ \mathrm{Re}\sum_{n=0}^{\infty}\sum_{n=0}^{\infty}(\lambda_{nm}-\lambda_n)^{-1}A_n(\lambda_{nm})\alpha_n(\lambda_{nm})R_n(\lambda_{nm}r)e^{-\lambda_{nm}z+in\vartheta} \\ \text{for (I) and (II);} \\ -4\ \mathrm{Re}\sum_{n=0}^{\infty}e^{in\vartheta}\Big[P\int_0^{\infty}(\lambda-\lambda_n)^{-1}A_n(\lambda)\alpha_n(\lambda)R_n(\lambda r)e^{-\lambda z}\,d\lambda + \\ +\pi i A_n(\lambda_n)\alpha_n(\lambda_n)R_n(\lambda_n r)e^{-\lambda_n z}\Big] \quad \text{for (III)} \end{cases} \tag{1}$$

(P denotes the principal value of the integral).

Here

$$A_n(\lambda) = \begin{cases} \lambda^2 \left[(\lambda^2 b^2 - n^2) R_n^2(\lambda b)\right]^{-1} & \text{for (I)}, \\ \lambda^2 \left[(\lambda^2 b^2 - n^2) R_n^2(\lambda b) - (\lambda^2 a^2 - n^2) R_n^2(\lambda a)\right]^{-1} & \text{for (II)}, \\ \lambda \left[J_n'^2(\lambda a) + N_n'^2(\lambda a)\right]^{-1} & \text{for (III)}; \end{cases}$$

$$R_n(\lambda r) = \begin{cases} J_n(\lambda r) & \text{for (I)}, \\ J_n(\lambda r) N_n'(\lambda a) - N_n(\lambda r) J_n'(\lambda a) & \text{for (II) and (III)}. \end{cases}$$

$$\alpha_n(\lambda) = \iint_S \gamma(r_1, \vartheta_1, z_1) R_n(\lambda r_1) e^{-\lambda z_1 - in\vartheta_1}\, dS,$$

$$\alpha_n(\lambda) = -\frac{\lambda}{2\pi} \int_0^{2\pi} \int_0^{\infty} \varphi_1(r, \vartheta, 0) R_n(\lambda r) e^{-in\vartheta} r\, dr\, d\vartheta \quad \text{for (III')},$$

$$\lambda_n = n^2 \omega^2 / g; \qquad \lambda_{nm} \text{ are the roots of } R_n'(\lambda b) = 0;$$

r, ϑ, z are cylindrical coordinates; r_1, ϑ_1, z_1 are the coordinates of points on the surface of the body S; a is the radius of the cylinder and of the inner wall of the channel; b is the radius of the basin and of the outer wall of the channel.

The expressions (1) give the exact value of the wave potential (under the assumptions made) at a sufficient distance from the body and, in particular, on the free surface of the fluid. Near the body they also give a good approximation, although they do not ensure strict satisfaction of the flow conditions. To obtain here the exact value, one must add to the potential φ (1) a term φ', determined from the condition $\partial\varphi'/\partial n = -\partial\varphi/\partial n$, for example, in the form

$$\varphi' = \iint_S \frac{\gamma_1}{R}\, dS, \qquad 2\pi\gamma_1 = \iint_S \gamma_1 \frac{\partial}{\partial n}\left(\frac{1}{R}\right) dS + \frac{\partial \varphi}{\partial n}. \tag{2}$$

The integral equation (2) has been well studied; it can be solved by the method of successive approximations.

The wave surface of the fluid $\zeta = -\dfrac{\omega}{g}\left(\dfrac{\partial\varphi}{\partial\vartheta}\right)_{z=0}$ in cases (I), (II), and also in the interior region ($r < r_1$) in case (III), will consist of waves running around a circle with the angular velocity of the body ω and having the form of standing waves in the coordinate r. In the exterior region ($r > r_1$), in case (III), at a sufficient distance from the trajectory of the body, the wave surface in absolute coordinates will have the form

$$\zeta = 4\left(\frac{2\pi}{r}\right)^{1/2} \frac{\omega}{g} \sum_{n=1}^{\infty} n \lambda_n^{1/2} A_n(\lambda_n) \left| \frac{\alpha_n(\lambda_n)}{H_n^{(1)}(\lambda_n a)} \right| \sin\left[\lambda_n r + n(\vartheta' - \omega t + \vartheta_n)\right],$$

$$n\vartheta_n = \arg\left[\alpha_n(\lambda_n) - H_n^{(1)'}(\lambda_n a)\right] - \frac{\pi}{2}\left(n - \frac{1}{2}\right).$$

Thus, in case (III), waves go off to infinity, whose fronts are Archimedean spirals of the form

$$r = -\frac{g}{n\omega^2}(\vartheta + \vartheta_n).$$

The components of the wave resistance in the cases under consideration are computed by the formulas

$$X = -\rho\omega \iint_S \varphi\beta\, dS, \qquad Y = \rho\omega \iint_S \varphi\alpha\, dS$$

(α, β are the direction cosines of the normal to S along the axes ox and oy). Let, for example, the body have the form of a sphere of radius c. Then, taking φ' into account, we obtain

$$\left.\begin{aligned} X &= -\frac{8\pi c^6\rho\omega^2}{l}\sum_{n=1}^{\infty}\sum_{m=0}^{\infty}(\lambda_{nm} - \lambda_n)^{-1} n^2 A_{1n}(\lambda_{nm}) R_n^2(\lambda_{nm} l) e^{-2\lambda_{nm} h}, \\ Y &= 0; \end{aligned}\right\} \quad \text{for (I) and (II)}$$

$$\left.\begin{aligned} X &= -\frac{4\pi c^6\rho\omega^2}{l}\sum_{n=1}^{\infty} n^2 P\int_0^{\infty}(\lambda - \lambda_n)^{-1} A_n(\lambda) R_n^2(\lambda l) e^{-2\lambda h}\, d\lambda, \\ Y &= -4\pi^2 c^6\rho\omega^2 \sum_{n=1}^{\infty} n\lambda_n A_n(\lambda_n) R_n(\lambda_n l) R_n'(\lambda_n l) e^{-2\lambda_n h} \end{aligned}\right\} \quad \text{for (III)}$$

($l, 0, h$ are the coordinates of the center of the sphere).

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Note: Figure translations are in progress. See original paper for figures.