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Abstract

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ON CAUSALITY IN NONLOCAL FIELD THEORY

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1. In connection with the increased capabilities of experimental technique, the question of the limits of applicability of the existing theory becomes topical. A natural generalization of the ordinary field theory is nonlocal field theory. A number of works have been devoted to this topic ([1,2]). However, it later turned out that attempts to combine the requirements of relativistic invariance, macrocausality, and unitarity encounter considerable difficulties ([3,4]). We shall show that these requirements can be combined in the second order of perturbation theory if the class of form factors is somewhat narrowed in comparison with that introduced in ([2]). Below the macrocausality of the model of field theory considered in ([1]) will be shown. It should be noted that this model is, generally speaking, nonunitary ([4,5]); however, it is quite probable that unitarity can be achieved by adding to the Lagrangian a number of terms containing closed loops of double lines (retarded functions), which vanish in the local limit ([4]). From what follows it will be clear that the results are not changed by such a modification of the Lagrangian.
2. As form factors we take functions $\Gamma(x, y, z)$, whose Fourier transforms have the form $\hat{\Gamma}(k_1, k_2, k_3) = \Gamma_1(k_1^2)\Gamma_2(k_2^2)\Gamma_1^*(k_3^2)$, where Γ_1, Γ_2 are meromorphic functions, analytic on the real axis. Their decrease at infinity is determined by the requirement of convergence of the corresponding integrals.

Following ([3]), we shall show the causality of the expression $\Gamma(D^cD^c - D^sD^s + \frac{1}{4}D^0D^0)\Gamma$. This will also constitute a proof of macrocausality of the second-order processes, since the other proofs are given in ([3]). Causality requires that, when the wave packets are separated in time, the terms containing frequencies of both signs decrease sufficiently rapidly.

A typical term of the expression under study is:

$$
\begin{aligned}
A = &\int \Gamma(x,y,z)\,[D_m^+(x-w)\theta(x-w)D_m(z-u) \\
&- D_m^+(x-w)D_m(z-u)\theta(z-u)]\Gamma(u,v,w)V_Y^*(y)V_V(v)\,dx\,dy\,dz\,du\,dv\,dw \sim \\
\sim &\int e^{i(V-Y)}\Lambda(k^2)\Lambda[(k+q)^2]H(q^2)\tilde V_Y^*(q)\tilde V_V(q) \\
&\times \left[\frac{D_m^+(p_0,-q-k)D_m^-(k)}{-q_0-k_0-p_0-i\varepsilon} - \frac{D_m^+(-q-k)D_m^-(p_0,k)}{k_0-p_0-i\varepsilon}\right] dq\,dk\,dp_0,
\end{aligned}
\tag{1}
$$

where $\Lambda = \Gamma_1\Gamma_1^*$, $H = \Gamma_2^2$; V_Y, V_V are wave packets with centers Y and V, respectively.

In what follows we shall regard the wave packets as finite sufficiently smooth functions, different from zero on $[Y-a/2, Y+a/2]$ and $[V-a/2, V+a/2]$, respectively. $\tilde V_Y$ and $\tilde V_V$ are their Fourier transforms with respect to–

variables $Y-u$ and $V-v$. Let $\tilde f(q) = \tilde V_Y^* \tilde V_V$; then $\tilde f$ is an entire function having the estimate

$$
\left|q_i^n \tilde f(q)\right| < B_n e^{a|\operatorname{Im} q_i|} \qquad (i = 1,\ 2,\ 3,\ 0); \tag{2}
$$

n is the order of smoothness of the packet, $B_n = \int |f^{(n)}(x)|\,dx$. For $X_0 = Y_0 - V_0 > a$ one can carry out the integration over q_0, closing the contour in the lower half-plane.

We shall assume that the form factors possess a minimal number of simple poles. In the case of multiple poles there will be no essential difficulties. The result will be represented as a sum of residues with respect to the poles of the form factors: $A = \sum_j A_j$.

Form the expression

$$
F_j^s = \int_a^\infty |A_j|\,X_0^s\,dX_0. \tag{3}
$$

F_j^s can be estimated from above. We give the estimate for the term corresponding to the residue at one of the poles H:

$$
\begin{aligned}
F_1^s < \text{const} \int &\left| \frac{\Lambda\left[m^2 + c_1^2 - 2\left(\sqrt{\mathbf{k}^2 - m^2}\sqrt{\mathbf{q}^2 + c_1^2} - \mathbf{qk}\right)\right]}{\sqrt{(\mathbf{q}^2 + c_1^2)(\mathbf{k}^2 + m^2)}\,[(\mathbf{q}+\mathbf{k})^2 - m^2]\left(\sqrt{\mathbf{k}^2 + m^2} - \sqrt{(\mathbf{k}+\mathbf{q})^2 - m^2} - \sqrt{\mathbf{q}^2 + c_1^2}\right)} \right| \times \\
&\times \sum_{l=0}^{s} \binom{s-l}{l} a^{s-l} \frac{l!}{\left|\operatorname{Im}\sqrt{\mathbf{q}^2 + c_1^2}\right|^{l+1}} \frac{B_n^2}{\left|\sqrt{\mathbf{q}^2 + c_1^2}\right|^{2n}}\,d\mathbf{q}\,d\mathbf{k},
\end{aligned}
\tag{4}
$$

where c_1^2 is the pole of H.

For $2n \geq s+1$ convergence of the right-hand side is ensured. If the wave packet has an infinite number of derivatives, the moments (3) exist for any s. As the causality condition we shall take the boundedness of F for any s.

3. Consider the scattering of a neutral scalar particle m by a charged scalar particle M, for which the interaction Lagrangian is

$$L_I = \int \Gamma(x', x'', x''')\, \varphi^*(x')\, U(x'')\, \varphi(x''')\, dx'\, dx''\, dx''' +$$

$$+ \int \Delta m\, U^2(x)\, dx + \int \Delta M\, \varphi^*(x)\varphi(x)\, dx; \qquad (5)$$

$U(x)$ is a neutral field with mass m, $\varphi(x)$ is a charged field with mass M.

The scattering matrix element is

$$T = \int \langle p'|\tilde{U}^-_{\text{out}}(k')\, \tilde{U}^+_{\text{in}}(k)|p\rangle\, \tilde{V}^{\text{out}}_X(k')\, \tilde{V}^{\text{in}}_Y(k)\, d\mathbf{k}\, d\mathbf{k}',$$

where $\langle p'|$, $|p\rangle$ are the states of the scatterer M before and after scattering; $V^{\text{out}}_X(x)$, $V^{\text{in}}_Y(y)$ are free wave packets. The dependence on the three-dimensional vector should be understood as

$$a(\mathbf{k}) = \left.\frac{a(k)}{\sqrt{2k_0}}\right|_{k_0=+\sqrt{\mathbf{k}^2+m^2}}.$$

Introducing four-dimensional variables and using the relation

$$U_{\text{out}}(x) = U_{\text{in}}(x) + \int D(x-x')\, j(x')\, dx',$$

we obtain

$$T = I + \frac{i}{2\pi} \int \left\langle \mathbf{p}' \left| \frac{\delta j(x)}{\delta U_{\text{in}}(y)} \right| \mathbf{p} \right\rangle e^{i(k'x-ky)}\, \tilde{V}_X(k')\tilde{V}^*_Y(k)\, dk\, dk'\, dx\, dy; \qquad (6)$$

$\tilde{V}_X$, $\tilde{V}_Y$ are Fourier transforms with respect to x and y.

For asymptotic states

$$I = \delta(\mathbf{p}-\mathbf{p}')\delta(K-K'),$$

$K,\ K'$ are the centers of the packets. We shall assume that the packets are chosen so that their widths $\Delta k_0/2,\ \Delta k_0'/2 \leq m$.

$$\left\langle \mathbf{p}' \left| \frac{\delta j(x)}{\delta U_{\text{in}}(y)} \right| \mathbf{p} \right\rangle = (\Box_x - m^2) e^{i(p'-p)x} \int e^{i(x-y)k} A_{p,p'}(k)\, dk,$$

where $A_{p,p'}(k)$ is the sum of the contributions of all G_N-diagrams of all orders (1)

$$A_{p,p'} = \int \prod_l dq_l \sum_{G_N} A_{G_N}(q,k) \sim$$

$$\sim \sum_{G_N} \int \prod_j D_R \left(k + \sum (\pm)_{jm} q_m\right) \left| \Gamma_b \left[\left(k + \sum_j (\pm)_{jm} q_m \right)^2 \right] \right|^2 \times$$

$$\times \prod_l D^-(q_l) \Gamma_2(k) \Gamma_2(k - p' - p) \Gamma_1(p') \Gamma_1^*(p) \prod_l dq_l; \qquad (7)$$

$(\pm)_{jm} = \pm 1$, the external momenta are included among the q_m, $b = 1, 2$.

For the study of causality we shall need the quantity

$$T^R = T - I \sim \int (k^2 - m^2) A(k) e^{i(p'-p)X} e^{ik(X-Y)} \tilde{V}_X(p' - p + k) \tilde{V}_Y^*(k)\, dk. \quad (8)$$

For $Z_0 = Y_0 - X_0 > a$, let us carry out the integration over k_0, closing the contour in the lower half-plane. Since the D_R have singularities only in the upper half-plane, the result will contain residues only with respect to the poles of the form factor. To avoid complications with multiple poles, we shall consider the case of forward scattering and diagrams without repeated lines. In the general case there will be no essential complications.

If we take $2n \geq s + 1$, then the moments

$$F_{G_N}^s = \int_{\bar{a}}^{\infty} \left| T_{G_N}^R \right| Z_0^s\, dZ_0$$

have the estimate

$$F_{G_N}^s < \text{const} \sum_{l,r} \frac{\bar{B}_n^2}{\bar{a}^{2n}} \sum_{m=0}^{s} \binom{s-m}{m} a^{s-m} m! \max_{k,q_j} \left[\frac{1}{\left|\operatorname{Im} k_0^{rl}\right|^{m+1}} - \frac{1}{\left|k_0^{rl}\right|^{2n}} \right] \times$$

$$\times \int dk \prod_l dq_l \left|(k^2 - m^2) A_{G_N}(q,k)\right|_{k_0 = k_0^{rl}}, \tag{9}$$

where

$$ck_0^{rl} = -\left(\sum_i (\pm)_{li} q_i^0\right) +$$

$$+ \left[\left(\sum_i (\pm)_{li} q_i^0\right)^2 + \mathbf{k}^2 - \left(\sum_i (\pm)_{li} \mathbf{q}_i\right)^2 + 2\mathbf{k}\left(\sum_i (\pm)_{li} \mathbf{q}_i\right) + \left(\sum_i{}' (\pm)_{li} q_i\right)^2 + c_r^2\right]^{1/2};$$

c_r^2 are the poles of $|\Gamma_b|^2$; $\bar{B}_n = B_n/a^n$; $\bar{a} = ac$; $c = \min_r |\operatorname{Im} c_r|$. Or, taking $2n = m+1$, we obtain

$$F^s_{G_N} < \mathrm{const} \cdot f^{G_N} \sum_{m=0}^{s} a^{s-2m-1} \frac{(2\sqrt{2})^{m+1} \overline{B}^2_{\frac{m+1}{2}}\, S!}{(s-m)!}, \tag{10}$$

where

$$f^{G_N} = \sum_{r,l} \int dk \left|(k^2 - m^2) A_{G_N}(q,k)\right|_{k_0 = k_0^{rl}} \prod_l dq_l.$$

In conclusion, I consider it my duty to express my gratitude to B. V. Medvedev for suggesting the topic and for supervising the work, and also to I. V. Polubarinov for discussing the work.

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Note: Figure translations are in progress. See original paper for figures.