HYDROMECHANICS

Corresponding Member of the Academy of Sciences of the USSR L. N. SRETENSKII

1960

SovietRxiv

Abstract

Full Text

HYDROMECHANICS

Corresponding Member of the Academy of Sciences of the USSR L. N. SRETENSKII

ON A HYDRODYNAMIC PROBLEM CONNECTED WITH THE TSUNAMI PROBLEM

The present article contains a summary of an investigation of one problem in the theory of the propagation of long waves in a rotating basin; we consider unsteady wave motions arising from certain initial disturbances, such as, for example, shocks accompanying submarine earthquakes and entailing the appearance of waves called “tsunami waves.” We have in mind investigating, under the simplest assumptions, the reflection of tsunami waves from a rectilinear shore.

Let us take the system of equations for the propagation of long waves in a basin of constant depth h; using the usual notation, we have:

$$\frac{\partial u}{\partial t} - 2\omega v = -g\frac{\partial \zeta}{\partial x}, \qquad \frac{\partial v}{\partial t} + 2\omega u = -g\frac{\partial \zeta}{\partial y}, \qquad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{1}{h}\frac{\partial \zeta}{\partial t} = 0.$$

The determination of the function $\zeta(x, y, t)$ reduces to the integration of two equations:

$$gh\Delta\zeta' - 4\omega^2\zeta' = -2\omega h\left[\frac{\partial v(0)}{\partial x} - \frac{\partial u(0)}{\partial y}\right] + 4\omega^2\zeta(0);$$

$$\frac{\partial^2 Z}{\partial t^2} + 4\omega^2 Z = gh\Delta Z, \qquad \zeta = \zeta' + Z.$$

The first equation, in which $u(0), v(0), \zeta(0)$ are the prescribed initial values of the required functions u, v, ζ, determines the steady part of the total elevation of the wave, while the second equation describes the unsteady part of the wave process proper; this part we shall find from the following initial and boundary data:

$$Z(x, y, 0) = \zeta(0) - \zeta'(x, y), \qquad \frac{\partial Z(x, y, 0)}{\partial t} = -h\left[\frac{\partial u(0)}{\partial x} + \frac{\partial v(0)}{\partial y}\right], \quad (1)$$

$$\left[2\omega\frac{\partial Z}{\partial x} - \frac{\partial^2 Z}{\partial t\,\partial y}\right]_{y=0} = 0;$$

the basin extends without bound in all directions from the coordinate axis $y = 0$ and corresponds to positive values of y.

The function $Z(x, y, t)$ is sought in the form of the following integral:

$$Z(x, y, t) = \int_{-\infty}^{\infty} A(t, y; k)e^{ikx}\,dk. \tag{2}$$

The function $A(t, y, k)$ will be an integral of the equation

$$\frac{\partial^2 A}{\partial t^2} - c^2\frac{\partial^2 A}{\partial y^2} + s^2 A = 0, \qquad s^2 = 4\omega^2 + c^2k^2, \qquad c^2 = gh,$$

satisfying the conditions

$$\left[\frac{\partial^2 A}{\partial t\,\partial y} - 2i\omega kA\right]_{y=0} = 0,$$

$$A(0, y, k) = f(y, k), \qquad \frac{\partial A(0, y, k)}{\partial t} = \varphi(y, k),$$

where $f(y, k)$ and $\varphi(y, k)$ are functions obtained by Fourier transformation from the right-hand sides of formulas (1).

Consider the function

$$B(t, y, k) = \frac{\partial^2 A}{\partial t\,\partial y} - 2i\omega kA;$$

this function satisfies the equation

$$\frac{\partial^2 B}{\partial t^2} - c^2\frac{\partial^2 B}{\partial y^2} + s^2 B = 0$$

and the conditions

$$B(t, 0, k) = 0, \qquad B(0, y, k) = \frac{\partial \varphi}{\partial y} - 2i\omega kf(y, k) \equiv m(y, k),$$

$$\frac{\partial B(0, y, k)}{\partial t} = c^2\frac{\partial^3 f}{\partial y^3} - s^2\frac{\partial f}{\partial y} - 2i\omega k\varphi \equiv n(y, k).$$

Applying Riemann' s method, we find the function $B(t, y, k)$:

$$B(t,y,k) = \frac{1}{2}[m(y-ct,k) + m(y+ct,k)] +$$

$$+\frac{1}{2c}\int_{y-ct}^{y+ct} \left[R(\sigma)n(\eta,k) + 2c^2 t R'(\sigma) m(\eta,k)\right] d\eta, \tag{3}$$

where $R = J_0\left(\frac{s}{c}\sqrt{\sigma}\right)$, $\sigma = c^2t^2 - (y-\eta)^2$.

Now the function $A(t, y, k)$ is determined by the formula

$$A(t,y,k) = F(t) + f(y,k) - f(0,k)S_0 + t\int_0^y f(\eta,k)S'(\rho_1)\,d\eta +$$

$$+y\int_0^t F(\tau,k)S'(\rho_2)\,d\tau + \int_0^t d\tau\int_0^y S(\rho)B(\tau,\eta,k)\,d\eta; \tag{4}$$

here $S(\rho)$ is the holomorphic integral of the equation

$$\rho\frac{d^2S}{d\rho^2} + \frac{dS}{d\rho} - 2i\omega kS = 0, \qquad S(0) = 1,$$

and

$$\rho_1 = t(y-\eta), \qquad \rho_2 = y(t-\tau), \qquad \rho = (\tau-t)(\eta-y), \qquad S_0 = S(ty).$$

The function $F(t)$, entering formula (4), is the solution of the equation

$$\frac{d^3F}{dt^3} + s^2\frac{d^2F}{dt^2} + 4\omega^2k^2c^2\int_0^t F(\tau)\,d\tau = 2i\omega kc^2\frac{\partial f(0,k)}{\partial y} + c^2\frac{\partial B(t,0,k)}{\partial y},$$

satisfying the conditions:

$$F(0) = f(0,k),$$

$$F'(0) = \frac{\partial\varphi(0,k)}{\partial y};$$

$$F''(0) = c^2\frac{\partial^2 f(0,k)}{\partial y^2} - s^2 f(0,k).$$

The determination of the function $F(t)$ causes no difficulty, and the set of formulas (2), (3), (4) will solve the posed problem of the reflection of unsteady long waves propagating in a rotating basin.

A complete exposition of the entire investigation, with analysis of various particular problems, will be given in *Izvestiya of the Academy of Sciences of the USSR*, Geophysical Series.

Marine Hydrophysical Institute
Academy of Sciences of the USSR

Received
14 XII 1959

Note: Figure translations are in progress. See original paper for figures.