ON CALCULATIONS OF FLOWS OF A GASIFIED LIQUID WITH A TWO-PARAMETER CHARACTERISTIC OF PERMEABILITIES

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Abstract

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ON CALCULATIONS OF FLOWS OF A GASIFIED LIQUID WITH A TWO-PARAMETER CHARACTERISTIC OF PERMEABILITIES

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In hydrodynamic calculations of flows of a gasified liquid, one usually uses relative permeabilities specified in the form of dependences $F_\mathrm{g}(\rho)$ and $F_\mathrm{l}(\rho)$, which are determined in experiments on the filtration of a mixture of liquid and gas introduced from outside. The data of new studies ([9]) show, however, that in steady filtration of a liquid saturated with gas, a substantially more complete description of the process is given by two-parameter dependences of the form

$$\frac{k_\mathrm{g}}{k} = F_\mathrm{g}(\rho, P^*); \qquad \frac{k_\mathrm{l}}{k} = F_\mathrm{l}(P^*, \lambda)$$

and by equivalent ones. Here k_g and k_l are the permeabilities for gas and for liquid; k is the initial permeability; ρ is the liquid saturation; $P^* = P/P_1$, where P_1 is the inlet pressure; $\lambda = \dfrac{\Gamma^*}{S(P)}$, where $S(P)$ and Γ^* are the volume solubility coefficient and the gas factor, reduced to pressure P_1.

In the present work the possibility of applying two-parameter dependences to the construction of flows is considered, and some examples are given.

1. Steady flows. The permeability during filtration of a gasified liquid is determined by the structure of the intrapore flow ([9]), which in turn may depend on the history of the motion, i.e., on the law of variation of the saturation and of the reduced pressure for an elementary volume. As a result, the admissibility of applying experimental data found in linear flow to flows with another geometry requires additional investigation. Specially designed experiments have shown that data found in linear flow can be used for approximate calculations of plane-radial flow. It should be assumed that satisfactory results will also be obtained for flows in a system of wells.

For a steady flow, when two-parameter permeabilities are introduced, the known solution ([5]) is preserved, reducing the problem to the construction of a potential flow. In the present case it is convenient to take $F_\mathrm{l}(\rho)$ and $F_\mathrm{l}(P^*, \lambda)$ as the dependences determining the permeability. In steady flow $\Gamma = \mathrm{const}$ and $P_1 =$

Figure 1

Figure 1: Figure 1

Figure 2

Figure 2: Figure 2

const. The quantity F_l is a function of one parameter $P^* = \text{const} \cdot P$. From the relation

$$H(P) = -\mu_l(P_n) \int_{P_n}^{P} \frac{F_l(P)\, dP}{\mu_l(P)\, \beta(P)} \tag{1}$$

the potential is found, and then also the other flow parameters ([5],[6]). Here μ_l is the viscosity and β is the volumetric coefficient of the liquid.

As an example, let us consider the calculation of indicator curves for the case of plane-radial inflow. Assume that the saturation pressure P_n is less than the contour pressure $P_k = \text{const}$. The bottom-hole pressure $P_c < P_n$. Using the condition of equality of flow rates in the regions $P > P_n$ and $P < P_n$, one can calculate the sought dependence $Q(\Delta P)$ from (4) (Fig. 2). The calculations were carried out for two oils with different physical properties*.

Permeability during the flow of a gasified liquid depends on the physical properties of the oils and gases. The viscosity ratio μ_g/μ_l has the most substantial influence. For oil A the permeability characteristics used were those given in ([9]); for oil B the characteristics are shown in Fig. 1.

Fig. 1. Permeability characteristics $F_l(P^*, \lambda)$. Uncemented sand, $\mu_g/\mu_l = 0.00090$, $S = 1.0$

Examination of the calculation results (Fig. 2) shows that the use of one-parameter dependences gives an overestimated flow rate. The difference in flow rates is most substantial for viscous oils.

Fig. 2. Dependence of the dimensionless flow rate
$q = Q\mu_l(P_n)/\pi h P_n k$ on the drawdown $\Delta P = P_k - P_c$.
1 —one-parameter permeability characteristic, 2 —two-parameter characteristic.
A —oil A, mean viscosity value $(\mu_l)_{av} = 2.3$ cP; B —oil B, $(\mu_l)_{av} = 20$ cP.
Contour radius $r_k = 250$ m, effective well radius $r_c = 10^{-3}$ m; $P_k = 165$ ata, $P_n = 140$ ata. Gas viscosity $\mu_g = 0.018$ cP.

2. Unsteady flows

In unsteady flow, specifying the saturation and the reduced pressure, generally speaking, does not determine the history of motion of an elementary volume, and in some cases it may differ radically from what occurs in steady-state flow.

Fig. 3

Figure 3: Fig. 3

* The properties of the oils were taken from ([1]).

flow. In those cases where these histories are sufficiently close, for calculations of unsteady flow one may use the permeability data obtained in steady flow; otherwise, additional investigation is required.

The use of steady-state permeabilities is possible, in particular, for plane-radial flow. Processing, in the Lagrangian representation, of experimental data on modeling the depletion of a circular reservoir ([2]) shows that in the vicinity of the well, where the principal pressure drop occurs, the histories of motion of elementary volumes are very close to the histories in steady flows. At a distance from the well, the permeabilities for the motion of a gasified liquid differ little from the permeabilities for the motion of a mixture ([9]). A comparison of the pressure distributions of steady and unsteady flows at equal production rates and gas factors shows an insignificant difference in the pressures at the boundary. Therefore, for calculations of plane-radial unsteady inflow one may apply permeabilities determined in steady flow, taking the boundary pressure $P_{\text{k}}(t)$ as the reference quantity. Such a choice of the reference quantity can introduce only a slight error into the determination of the permeabilities of the near-wellbore zone. For the entire remaining region $P^* \simeq 1$, i.e., the average-reservoir permeabilities practically coincide with the permeabilities for the mixture, which agrees with the results of field and laboratory investigations ([2], [10]).

Fig. 3

The dimensionless equations of plane-radial flow have the form

$$\frac{1}{r}\left\{\frac{\partial}{\partial r}\left[\frac{z(P)F_{\text{g}}(\rho,P^*)}{\mu_{\text{g}}(P)}r\frac{\partial P}{\partial r}\right]+\frac{\partial}{\partial r}\left[\frac{S(P)F(\rho)}{\mu\,(P)\beta(P)}r\frac{\partial P}{\partial r}\right]\right\}=\frac{\partial}{\partial t}\left[z(P)(1-\rho)+\frac{S(P)}{\beta(P)}\rho\right];$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left[\frac{F(\rho)}{\mu\,(P)\beta(P)}r\frac{\partial P}{\partial r}\right]=\frac{\partial}{\partial t}\left[\frac{\rho}{\beta(P)}\right]. \qquad (2)$$

The dimensionless quantities entering equations (2)—radius r, time t, pressure P, viscosity μ, and the quantity z—are defined by the relations

$$r=\frac{r_d}{(r_{\text{k}})_d},\qquad t=t_d\frac{k_d(P_{\text{k}})_d}{[\mu\,(P)]_d(r_{\text{k}})_d^2m},\qquad P=\frac{P_d}{(P_{\text{k}})_d},$$

$$\mu=\frac{\mu_d}{[\mu\,(P)]_d},\qquad z=\frac{[\gamma_{\text{g}}(P)]_d(P\ \ )_d}{[\gamma_{\text{g}}(P\ \ )]_dP_d},$$

where m is porosity, γ_{g} is the volumetric weight of the gas, and r_{k} is the boundary radius. The subscript d indicates that the quantity is dimensional.

Equations (2) have the same form as the equations for one-parameter permeability characteristics ([10], [6], [3]). The essential difference, however, is that, since $P^* = P/P_{\mathrm{k}}$, the coefficients of equations (2) contain the function $P_{\mathrm{k}}(t) = P(r,t)|_{r=1}$, which is not known in advance.

Let us consider the depletion of a circular reservoir under a dissolved-gas drive regime. At the boundary, where $P^* = 1$ and the function $\Psi(\rho, P^*) = F_{\mathrm{g}}(\rho, P^*)/F(\rho)$ coincides with the function $\Psi(\rho)$ of the mixture, the meaning and quantitative results of the solutions ([6], [3]), which establish the dependence $P(\rho)$ on an impermeable equipotential, remain valid. For the flow as a whole, a solution of system (2) without additional assumptions can be obtained only numerically.

A more satisfactory approximation in this case is given by the method of successive replacement of steady states ([6,7]). The results obtained by this method for calculating the depletion process at constant bottom-hole pressure are given in Fig. 4. The dependences $P_k(\rho_k)$ (Fig. 3) and $\Gamma^*(\rho_k)$ were found by the method described in ([8]). Next, for a number of values $\lambda = \text{const}$, according to (1), the function of S. A. Khristianovich was calculated, the production rate was determined, and the depletion equation was applied.

Fig. 4. Change of production rate with time in the depletion process.
1—one-parameter permeability characteristic, 2—two-parameter.
A—oil A, $P_n = 140$ atm, $P_c = 100$ atm; B—oil B, $P_n = 150$ atm, $P_c = 100$ atm.
The physical properties of the oils and gas are the same as for Fig. 2.

Consideration of the results shows that the use of one-parameter dependences for the permeabilities leads to an overestimation of the production rate in the first period and to an underestimation of the development time.

In conclusion, let us note that two-parameter dependences for the permeabilities can also be applied to the self-similar solution of the problem of the flow of a gas-saturated liquid toward a point sink in an infinite reservoir ([3]). Here the introduction of two-parameter permeabilities does not introduce additional computational difficulties, since the reduced quantity $P_k = P(r,t)|_{r=\infty} = \text{const}$, and the coefficients are known functions of pressure, saturation, and the dimensionless coordinate.

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Note: Figure translations are in progress. See original paper for figures.