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Fig. 1

Figure 1: Fig. 1

Abstract

Full Text

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PHYSICS

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CALCULATION OF PHASE TRAJECTORIES OF CHARGED PARTICLES WITH ALLOWANCE FOR COULOMB INTERACTION IN THE BUNCHER OF A LINEAR ELECTRON ACCELERATOR

(Presented by Academician I. V. Obreimov, 16 VI 1960)

Because of the exceptional complexity of the dynamical equations for a large number of charged particles, direct integration of these equations has not been carried out, and many-particle problems have been solved by methods of statistical physics([1,2]). Modern high-speed electronic machines make it possible to return to the direct method of solving the many-particle problem, formulating it as a Cauchy problem with initial conditions.

Fig. 1. Example of the dependence $E(\varphi\ \ )$ and $\varphi(\varphi\ \ )$ for $ct = 60$; $I = 0.1$ a (1). For comparison, the same dependence is shown for $I = 0$ (2)

In the present work this method is illustrated by the example of calculating the longitudinal motion of an electron beam, with allowance for their interaction, in a linear accelerator in the relativistic approximation. It should be noted that previously([3,4]), in calculating accelerators, the interaction of charges was not taken into account; however, in accelerators with large current, as will be seen below, it has a substantial influence on particle bunching and on their capture into the acceleration regime.

The initial equations of the longitudinal motion of the j-th charged particle under the action of the electric field of a traveling wave([3]) have the form

Figure 2 plot

Figure 2: Figure 2 plot

$$\frac{d\varphi_j}{dz} = k\left(\frac{1}{\beta\,(z)} - \frac{1}{\sqrt{1-(E_0/E_j)^2}}\right), \tag{1}$$

$$\frac{dE_j}{dz} = e\varepsilon_0(z)\sin\varphi_j + e\Delta F_z, \qquad j = 1, 2, \ldots, N,$$

where e is the particle charge; k is the wave vector of free space; φ_j is the phase of the j-th particle relative to the wave phase; $0 \le \varphi_j(0) \le 2\pi$; β —pha-

the phase velocity of the wave as a prescribed function of z; E_j is the energy of the j-th particle; $E_0 = m_0c^2$ is the rest energy.

The term ΔF_z takes into account the interaction of the given particle with the others, and it can be represented by means of retarded potentials [5]

$$\Delta F_z = \sum_{i,\ i\neq j}\left\{\frac{e_i}{s_i^3}z_{0i}(1-\dot{z}_i^2) - \frac{e_iR_i\ddot{z}_i}{s_i^2} + \frac{e_iR_i\dot{z}_i\ddot{z}_i}{s_i^3}\left(\frac{z_i}{R_i} - \dot{z}_i\right)\right\}; \tag{2}$$

$$s_i = \sqrt{r_i^2(1-\dot{z}_i^2) + z_i^2}, \qquad z_{0i} = z_i - \dot{z}_{iR}i,$$

where all quantities on the right-hand sides of the equality are referred to the time instant $\tau_i = t_i - R_i/c$; the summation is carried out over all particles; R_i is the distance from the observation point to the charge; $\dot{z}_i = dz_i/c\,dt$, and $\ddot{z}_i = d^2z_i/c^2dt^2$.

It is required to find the solution $\varphi(z)$ and $E(z)$ of equation (1) under prescribed initial conditions; for example, at $z = 0$

$$\varphi_j = \frac{2\pi}{N}j, \qquad E_j = \text{const} > E_0,$$

$$j = 1, 2, \ldots, N, \tag{2'}$$

and the beam has a finite diameter r_0, with particles uniformly distributed over the cross section. We simplify this problem by taking into account that, in the initial section of the buncher, the retardation effect may be neglected. Therefore, replacing e_i in (2) by $\rho\,dV$ and passing from the sum to an integral, we obtain

Fig. 2. Example of the dependence $\varphi(\varphi_{\text{init}})$ for 10 divisions (1) and 15 divisions (2). The line for 20 divisions practically coincides with 2.
$ct = 40$; $I = 0.1$ A.

$$\Delta F_z = \int_V \left[\frac{z_0(1-\dot{z}^2)}{s^3} + \frac{R\ddot{z}}{s^3}\left(\frac{z}{R} - \dot{z}\right) - \frac{R\ddot{z}}{s^2} \right] \rho \, dV, \tag{3}$$

where all quantities under the integral are referred to the time instant t. Expression (3) is further simplified somewhat because of the transverse homogeneity of the beam. In addition, for a sufficiently small integration step one may average $\dot{z}$ under the integral sign and set $\ddot{z} = 0$.

We replace z by $\alpha\varphi$, where $\alpha = \beta_{\text{w}}/k$, and, using the obvious equality

$$\rho = c_0 \sum \Delta\varphi_{\text{init}} = c_0 \sum \frac{\Delta\varphi(t)}{\varphi(\varphi_{\text{init}})}$$

(here c_0 is a constant determined by the initial charge density), we represent (3) in the form

$$\Delta F_z = \int \frac{\alpha[\varphi(\varphi_{\text{init}}) - \varphi_j(\varphi_{\text{init}})](1-\dot{z}^2)\, r\, dr\, d\varphi_{\text{init}}}{\left\{[\varphi(\varphi_{\text{init}}) - \varphi_j(\varphi_{\text{init}})]^2 - (1-\dot{z}^2) r^2 \alpha^2\right\}^{3/2}}. \tag{4}$$

Substituting ΔF_z into (1) and passing to the independent variable ct, we obtain a system of ordinary differential equations

$$\frac{dE_j}{c\,dt} = e\dot{z}_j \varepsilon_0 \sin\varphi_j + 2\pi e \dot{z}_j \int \frac{\alpha[\varphi - \varphi_j](1-\dot{z}^2)\, r\, dr\, d\varphi}{\left[(\varphi-\varphi_j)^2 + (1-\dot{z}^2) r^2 \alpha^2\right]^{3/2}}, \tag{5}$$

$$\frac{d\varphi_j}{c\,dt} = \beta_0 \dot{z}_j - k, \qquad \beta_0 = \frac{k}{\beta_{\text{w}}}, \qquad \dot{z}_j = \sqrt{1-(E_0/E_j)^2}, \qquad j = 1, 2, \ldots, N,$$

the solution of which is sought under the initial conditions (2′) and $t = 0$, while the integration region (4) is divided into N intervals*. It is assumed that as $N \to \infty$ the system (5) has a solution approaching the solution of (1) under the corresponding simplifications (2).

The system was solved on the BESM-1 of the Academy of Sciences of the USSR for several cases of $\beta_v(z)$, $\varepsilon_0(z)$, and $\varepsilon_{0\,\max}$ encountered in practice, and for the particle current.

Fig. 3. Example of the dependence $\varphi(\varphi_{\text{initial}})$ for 10 subdivisions (1), 15 subdivisions (2), 20 subdivisions (3) (line for 25 subdivisions practically coincides

Fig. 3

Figure 3: Fig. 3

with 3). $ct = 25$; $I = 1.0$ a. For comparison, the same dependence is shown for $I = 0$ (4).

The results of the calculation may be briefly summarized as follows:

1. The number of subdivisions must be sufficiently large (see Fig. 1).
2. In accelerators with small currents (of the order of 0.1 a and less), for the usually encountered parameters the influence of space charge can be neglected (see Fig. 2).
3. In accelerators with large currents (of the order of 1.0 a), it is necessary to take into account the Coulomb interaction both on the bunching of particles and on their capture into the acceleration regime (the outer particles are lost). The number of subdivisions must be increased to 25 (see Fig. 3).
4. The influence of neighboring bunches $\varphi(0) > 2\pi$, $\varphi(0) < 0$ on particle bunching can in any case be neglected.
5. Decreasing $d\beta_v(z)/dz$, increasing $\varepsilon_{0\,\mathrm{max}}$, and $d\varepsilon_0(z)/dz$ reduce the influence of the Coulomb interaction.

This method can also be used in calculating radial oscillations in a betatron and in a linear proton accelerator.

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Moscow Engineering Physics Institute

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* The singularity at the point $\varphi = \varphi_i,\ r = 0$, is defined as the principal value of an improper integral.

Note: Figure translations are in progress. See original paper for figures.