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Abstract

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"GHOST" STATES AND THE CONDITION OF CROSSING SYMMETRY

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1. Introduction. It has been suggested ([1]) that the problem of "ghost" states arises as a consequence of neglecting the condition of crossing symmetry. This suggestion was based on results obtained for the Lee model ([2]), which does not possess crossing symmetry, and for charged scalar meson theory with a fixed source ([3]), which does possess crossing symmetry. We shall construct a model with a fixed source that has an exact solution and, although it possesses crossing symmetry, nevertheless can have "ghost" states for sufficiently large values of the renormalized coupling constant g^2.

2. Model. A fixed source—a fermion (we shall call it a nucleon)—interacts with the field of relativistic charged (plus or minus) fermions (we shall call them χ-particles). The nucleon has two states, proton and neutron, differing by charge. The Hamiltonian of the model is

$$H = H_0 + H'; \tag{1}$$

$$H_0 = \sum_{(k)} \omega_k \{a^+_{k,+} a_{k,+} + a^+_{k,-} a_{k,-}\}; \tag{2}$$

$$H' = g_0 \sum_{(k)} v(k)(2\omega_k)^{-1/2} \{(a^+_{k,+} + a_{k,-})A + A^+(a_{k,+} + a^+_{k,-})\}, \tag{3}$$

where $\omega_k = \sqrt{k^2 + \mu^2}$ (μ is the mass of the χ-particle); g_0 is the unrenormalized coupling constant; $a_{k,\pm}$ ($a^+_{k,\pm}$) are the annihilation (creation) operators of a χ-particle with momentum k and charge sign $\pm$. The operator A^+ (A) transforms a neutron (proton) into a proton (neutron). Here $v(k)$ is a symmetric, real function depending on the form of the nucleon (momentum cutoff). The commutation rules for the operators are:

$$AA^+ + A^+A = 1; \tag{4}$$

$$a_{k,+}a^{+}_{q,+} + a^{+}_{q,+}a_{k,+} = a_{k,-}a^{+}_{q,-} + a^{+}_{q,-}a_{k,-} = \delta_{kq}. \tag{5}$$

All other pairs of operators anticommute. Our model has the property of crossing symmetry, since the interaction Hamiltonian (3) is invariant under the transformation:

$$a_{k,+} \to a^{+}_{-k,-}; \qquad a_{k,-} \to a^{+}_{-k,+}; \qquad a^{+}_{k,+} \to a_{-k,-}; \qquad a^{+}_{k,-} \to a_{-k,+}.$$

3. Green' s functions. Following the method proposed in ([4]), let us consider in our model the retarded and advanced Green' s functions:

$$\langle\!\langle A(t) \mid A^{+}(t') \rangle\!\rangle_{\text{ret}} = \theta(t-t')\langle (A(t)A^{+}(t') + A^{+}(t')A(t))\rangle; \tag{6}$$

$$\langle\!\langle A(t) \mid A^{+}(t') \rangle\!\rangle_{\text{adv}} = -\theta(t'-t)\langle (A(t)A^{+}(t') + A^{+}(t')A(t))\rangle. \tag{7}$$

where $\langle ... \rangle$ denotes averaging over one-nucleon states; $A(t)$ are operators in the Heisenberg representation.

For the Green' s functions (6) and (7) we write a chain of equations:

$$i\,\frac{d}{dt}\,\langle\langle A(t) \mid A^{+}(t')\rangle\rangle^{\text{ret}}_{\text{adv}} = i\delta(t-t') + \langle\langle i\,\frac{dA(t)}{dt} \mid A^{+}(t')\rangle\rangle^{\text{ret}}_{\text{adv}}, \tag{8}$$

where the value of the derivative of the Heisenberg operator $A(t)$ is found from the equations of motion.

We introduce the Fourier transforms of the Green' s functions

$$\langle\langle A(t) \mid A^{+}(t')\rangle\rangle^{\text{ret}}_{\text{adv}} = \int_{-\infty}^{+\infty} \langle\langle A \mid A^{+}\rangle\rangle^{\text{ret}}_{\text{adv}}(E) e^{-iE(t-t')}\,dE, \tag{9}$$

which will be analytic functions: the retarded one in the upper half-plane and the advanced one in the lower half-plane of E as a complex variable.

We consider $\langle\langle A \mid A^{+}\rangle\rangle_{\text{ret}}(E)$ and $\langle\langle A \mid A^{+}\rangle\rangle_{\text{adv}}(E)$ as a single function $\langle\langle A \mid A^{+}\rangle\rangle(E)$ and write for it a chain of equations

$$E\langle\langle A \mid A^{+}\rangle\rangle(E) = \frac{i}{2\pi} + g_0 \sum_{(k)} v(k)(2\omega_k)^{-1/2}\langle\langle (a_{k,+} + a^{+}_{k,-}) \mid A^{+}\rangle\rangle(E); \tag{10}$$

$$(E-\omega_k)\langle\langle a_{k,+} \mid A^{+}\rangle\rangle(E) = g_0 v(k)(2\omega_k)^{-1/2}\langle\langle A \mid A^{+}\rangle\rangle(E); \tag{11}$$

$$(E+\omega_k)\langle\langle a^+_{k,-} \mid A^+\rangle\rangle(E) = g_0 v(k)(2\omega_k)^{-1/2}\langle\langle A \mid A^+\rangle\rangle(E). \tag{12}$$

Thus, in our model the chain of equations closes and the Green' s function can be found exactly:

$$g_0^2\langle\langle A \mid A^+\rangle\rangle(E) = \frac{i}{2\pi} g^2 \frac{1}{E}\left[1 + g^2 J(E)\right]^{-1}. \tag{13}$$

Here a charge renormalization has been performed

$$g^2 = g_0^2(1 - g^2 N); \tag{14}$$

$$N = \sum_{(k)} v^2(k)\frac{1}{\omega_k^3}; \qquad J(E) = \sum_{(k)} v^2(k)\frac{E^2}{\omega_k^3(\omega_k^2 - E^2)}. \tag{15}$$

The Green' s function obtained has an additional pole at a certain purely imaginary value of E, if the renormalized coupling constant g^2 is greater than the critical value: $g_{\mathrm{cr}}^{-2} = N$. Thus, we arrive at the conclusion that our model, despite the fact that it satisfies the condition of crossing symmetry, will have a “ghost” state if $g^2 > g_{\mathrm{cr}}^2$.

4. Low equation for scattering. Let us note the connection of the Green' s functions obtained with the solutions of the Low equations [(5)] for scattering. In our model the Low equations for scattering of χ-particles by a proton in the one-particle approximation have the form

$$\frac{f_\alpha(k)}{kv^2(k)} = -\frac{g^2}{4\pi}\frac{1}{\omega_k} + \frac{1}{\pi}\int_\mu^\infty \left[\frac{|f_\alpha(q)|^2}{\omega_q - \omega_k} - \frac{|f_\beta(q)|^2}{\omega_q + \omega_k}\right]\frac{d\omega_q}{qv^2(q)}, \tag{16}$$

where $\alpha = 0, 1$; $\beta = 1 - \alpha$ ($\alpha = 0$ for a positive χ-particle; $\alpha = 1$ for a negative χ-particle); $f_\alpha(k) = \sin[\delta_\alpha(k)]\exp[i\delta_\alpha(k)]$ is the scattering amplitude of χ-particles with momentum k by a proton. In deriving (16) we con-

that the system consisting of a nucleon plus the field of χ-particles has no bound states with energy less than the energy of the proton state $E_p = 0$, and has only one bound state in the energy interval $0 \le \omega_k < \mu$, namely the neutron state with energy $E_N = E_p = 0$. The solutions of the Low equations (16) can be obtained by means of the well-known procedure used in paper ([3]). However, in our model the Green' s functions

$$g_0^2\langle\langle A \mid A^+\rangle\rangle(\omega_k) = -i\,\frac{f_0(k)}{kv^2(k)}; \tag{17}$$

$$g_0^2\langle\langle A^+ \mid A\rangle\rangle(\omega_k) = -\,i\,\frac{f_1(k)}{kv^2(k)} \tag{18}$$

give solutions of the Low equations (16). In view of what was said in Sec. 3, the obtained solutions (17) and (18) of the Low equations (16) will have an extraneous pole at a certain purely imaginary value of E, if $g^2 > g^2_{\mathrm{cr}}$.

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Note: Figure translations are in progress. See original paper for figures.