Reports of the Academy of Sciences of the USSR

S. G. MAKOVER and N. A. BOKHAN

1960

SovietRxiv

Abstract

Full Text

Reports of the Academy of Sciences of the USSR
1960. Volume 134, No. 3

ASTRONOMY

S. G. MAKOVER and N. A. BOKHAN

THE MOTION OF ENCKE-BACKLUND'S COMET IN THE YEARS 1898–1911 AND A NEW DETERMINATION OF THE MASS OF MERCURY

(Presented by Academician V. G. Fesenkov on 23 III 1960)

Encke-Backlund' s comet, discovered by Pons in Marseille in 1818, belongs to the group of short-period comets of Jupiter' s family. Its period of revolution is 3.3 years, and its perihelion distance is 0.34 AU. Owing to these features of its orbit, the comet can approach Mercury more closely than any other celestial body known to us and, consequently, can be used to determine the mass of this planet.

Previous investigators of the comet' s motion (Encke, Asten, and Backlund) repeatedly attempted to determine the mass of Mercury. However, the values of the mass obtained by them varied within very wide limits, from 1 : 2,668,700 to 1 : 10,252,900. The mean of several determinations, obtained in 1894 by Backlund ([1]) and used by him in all his subsequent works, is equal to 1 : 9,700,000; this value differs considerably from the modern value (1 : 6,000,000).

These considerable discrepancies are connected with the fact that the motion of Encke-Backlund' s comet cannot be fully explained by Newton' s law of gravitation. In the comet' s motion there is observed a secular acceleration of the mean motion and a secular decrease of the eccentricity, apparently caused by the action of little-studied reactive forces on the comet' s nucleus. Therefore the weak perturbing action of Mercury on the comet is difficult to separate from the action of the secular acceleration.

Another difficulty in earlier determinations of the mass of Mercury consisted in the necessity of allowing, with the required degree of accuracy, for the planetary perturbations acting on the comet. It is clear that the perturbations from Jupiter (and from other planets) must be computed with an error small in comparison with the perturbing action of Mercury. But the perturbations produced by Mercury in the comet' s mean anomaly reach, at the epochs of closest approaches, 50″ per revolution of the comet, whereas the perturbations from Jupiter often exceed 1°. Therefore these perturbations must be computed with great relative accuracy. It is especially important that small changes in the orbit, resulting from improvement of its elements, cause noticeable changes in the perturbations; hence, after each improvement of the orbit, the perturbations must be recomputed. However, in Backlund' s time, because of the low level of computational technique, such repeated recomputation of the perturbations was not feasible. First of all, the perturbations from the inner planets were computed on the outer part of the comet’ s orbit not by the method of mechanical quadratures, but from approximate analytic expressions, and these expressions were constructed on the basis of certain mean values of the comet’ s elements, different from the osculating ones. Furthermore, perturbations of the second and higher orders—generally speaking, not small—were not always computed. Moreover, a rigorous account of second-order perturbations was altogether impossible because of the peculiarities of the computation of perturbations by the inner planets. And, finally, the values

the masses of the perturbing planets differed appreciably from the modern ones; in particular, the masses of Mars and Mercury were in error by 50%.

Baklund^(2) encountered especially great difficulties in treating the motion of the comet for the years 1895–1908. During this period the comet had a close approach to Jupiter (1903; $\Delta_{\text{min}} = 0.9$) and to Mercury (1905; $\Delta_{\text{min}} = 0.056$). The residual deviations of the normal place for 1908 proved inadmissibly large: $\Delta\alpha = +250''.5$; $\Delta\delta = +122''.8$. In the final adjustment Baklund had to discard this normal place.

In 1954 and 1956 a work by S. G. Makover^(3) was published, devoted to an investigation of the comet’ s motion for the years 1937–1954. This period is of special interest because of the comet’ s close approach to Mercury, which took place in 1947 ($\Delta_{\text{min}} = 0.042$). The work showed that, after a careful allowance for planetary perturbations, it is possible to represent all the normal places of the comet with errors not exceeding the admissible errors of the observations. In the same work a new value of the mass of Mercury was obtained for the first time, in good agreement with determinations based on other methods:

$$m = 1 : (6\,280\,000 \pm 350\,000).$$

At present a new attempt to determine the mass of Mercury has been completed, using five apparitions of the comet closest to the epoch of the 1905 approach, namely the years 1898–1911. As in the preceding work, special attention was paid to a rigorous allowance for the perturbations, including their repeated recomputation after improvement of the orbit. The results of these computations are given in Tables 1 and 2.

Table 1

Normal places and residual errors

No. p.p.	Date (mean Greenwich time)	$\alpha_{1900.0}$	$\delta_{1900.0}$	Number of observations*	$\Delta\alpha$	$\Delta\delta$
1	1898 June 13.0	$6^h58^m07^s45$	$+10°22'02''.3$	1–3	$-0''.9$	$0''.0$
2	1901 August 11.0	6 36 42.72	+31 16 14.0	30–33	+6.4	−2.1
3	20.0	7 44 20.32	+28 38 59.2	33–47	+2.8	−0.7
4	September 2.0	9 29 03.83	+19 38 13.5	5–6	−6.6	+4.2
5	1904 November 9.0	22 42 15.34	+21 28 51.5	16–14	−0.1	−0.2
6	December 4.0	21 02 17.75	+7 49 29.2	65–69	−4.5	−1.9
7	19.0	20 00 32.27	−2 21 28.7	54–51	+2.3	+2.1
8	1908 May 28.0	2 57 20.35	−7 56 26.0	2–2	−1.1	+1.6
9	1911 July 31.65465	7 27 13.10	+26 55 18.9	1–1	−0.7	−4.2
10	September 7.0	12 30 21.28	−9 42 05.7	9–9	+3.1	−0.9
11	24.0	14 25 22.46	−22 21 31.9	3–3	−6.7	−1.8

* Separately for each coordinate.

Table 2

Final values of the elements

Epoch and osculation	M_0	ω	Ω	i	φ	μ
		ecliptic 1900.0				
1898 June 15.0	$5^\circ.70709$	$184^\circ.00102$	$334^\circ.79095$	12.90863	$57^\circ.82252$	0.29838525
1901 July 26.0	344.65264	.00356	.78284	.89291	.78398	22175
1904 November 15.0	342.71534	.63676	.34928	.57345	.90040	63004
1908 May 26.0	7.49777	.63918	.34764	.58817	.94446	84254
1911 June 28.0	344.45200	.66992	.32800	.57393	.91551	79968
September 8.0	5.96610	.67056	.32718	.57388	.91393	82842

From Table 1 it is seen that all the normal places are satisfactorily represented by the elements. In the last adjustment a system of 22 conditional equations with 9 unknowns was solved (six corrections to the elements, corrections to the secular acceleration x and to the secular change of the eccentricity angle φ', and also a correction to the mass of Mercury). For the secular changes of the elements, there were obtained—

determined values: $\chi = 27''.6$; $\varphi' = -3''.3$. Finally, the new value of the mass of Mercury was found to be $m = 1 : (5\,880\,000 \pm 200\,000)$.

We see that the two determinations of the mass of Mercury (from the 1947 and 1905 approaches) agree well with one another within their probable errors, but contradict Backlund' s determination $m = 1 : 9\,700\,000$. If from these two independent determinations a weighted mean is formed, then finally we obtain: $m = 1 : (5\,980\,000 \pm 170\,000)$ (probable error).

Table 3

Modern determinations of the mass of Mercury (with their probable errors)

Source	Year	Method	m
([4])	1949	Motion of the perihelion of the Earth	$1:(6\,400\,000 \pm 200\,000)$
([5])	1950	Perturbations of Eros	$1:(6\,120\,000 \pm 43\,000)$
([6])	1956	Perturbations of Venus	$1:(5\,970\,000 \pm 450\,000)$
Makover and Bokhan	1960	Comet Encke–Backlund	$1:(5\,980\,000 \pm 170\,000)$

In conclusion we give a summary of modern determinations of the mass of Mercury.

Institute of Theoretical Astronomy
Academy of Sciences of the USSR

Received
17 III 1960

CITED LITERATURE

[1] O. Backlund, Bull. Astr., 11 (1894).
[2] O. Backlund, Zap. AN, ser. 8, 23 (1908); 24 (1909); 30 (1911).
[3] S. G. Makover, Tr. ITA, 4 (1954); 6 (1956).
[4] G. M. Clemence, Proc. Am. Phil. Soc., 93, No. 7 (1949).
[5] E. Rabe, Astr. Journ., 55, No. 4 (1950).
[6] R. L. Duncomb, Astr. Journ., 61, No. 6 (1956).

Note: Figure translations are in progress. See original paper for figures.