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Fig. 1. Diagram of a fractured porous medium

Figure 1: Fig. 1. Diagram of a fractured porous medium

Abstract

Full Text

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ON THE FUNDAMENTAL EQUATIONS OF FILTRATION OF HOMOGENEOUS FLUIDS IN FRACTURED ROCKS

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1. In the present work a formulation is given of the problem of filtration of homogeneous fluids in rocks with strongly developed fracturing. At present a number of studies have been carried out on filtration in strata in the presence of a small number of fractures arranged in various ways. However, an approach based on a concrete accounting for the configuration of fractures in a stratum is not applicable to media with developed fracturing. Indeed, on the one hand, this configuration is unknown to us, and on the other hand it is extremely intricate and complex, so that the mathematical difficulties of calculating filtration for such complicated fracture configurations, even if they were known, become insurmountable. However, it is precisely the complexity and ramification of the systems of fractures in rocks with strongly developed fracturing that makes appropriate for such rocks another treatment, specific to continuum mechanics.

A porous rock with strongly developed fracturing may be represented as the superposition of two porous media with pores of different scales (Fig. 1). Let us imagine mentally that the boundary between the fractures and the blocks has become impermeable. Then the motion of the fluid in the porous medium will occur separately through the system of fractures separating the blocks (medium 1), and through the system of blocks communicating with one another (medium 2). Medium 1 may be likened to an ordinary porous medium in which the role of pore channels is played by fractures, and the role of grains by blocks. (In individual cases the area of contact of the blocks with one another may prove to be very small, so that, with an impermeable boundary between the fractures and the blocks, motion in medium 2 will not occur.) A characteristic feature of motion in fractured rocks is that in reality an intensive exchange of fluid takes place between the two media.

Fig. 1. Diagram of a fractured porous medium

In accordance with natural ideas about rocks with developed fracturing, under such a treatment it is assumed that the size of the blocks separated by fractures is small in comparison with the characteristic size of the given volume of porous rock, so that each physically infinitesimal volume contains a large number of blocks. Thus, at each point of space one may consider two pressures and two filtration velocities of the fluid: the pressure p_1 and the velocity $\mathbf{V}_1$ of the fluid in medium 1 and the pressure

p_2 and the velocity $\mathbf{V}_2$ of the fluid in medium 2. Here p_1 is the mean fluid pressure in the **fractures** in a neighborhood of the given point, and p_2 is the mean pressure in the **blocks** in a neighborhood of the given point. The velocity $\mathbf{V}_1$ is a vector whose projection on some given direction is equal to the flux of fluid flowing through the cross-sections of fractures of a small area passing through the given point perpendicular to this direction, divided by the full area of the small surface and by the fluid density. Similarly, the projection of the velocity vector $\mathbf{V}_2$ on the given direction is equal to the flux of fluid passing through the cross-sections of the **blocks** of the above-mentioned small surface, also divided by the full area of the surface and by the fluid density.

It is important to bear in mind that m_1—the porosity of the first medium, i.e., the ratio of the volume of fractures to the volume of the entire rock—is usually very small (0.001-0.01) and is one or even several orders of magnitude smaller than the porosity of the second medium m_2. At the same time, the permeability k_1 of the first medium exceeds the permeability of the second medium k_2 by several orders of magnitude [1-3,5].

2. Assuming the motion in both media to be inertia-free, one can show that the pressure drop in each medium is related to the filtration velocity by a linear relation—Darcy' s law:

$$\mathbf{V}_1 = -\frac{k_1}{\mu}\,\mathrm{grad}\,p_1, \qquad \mathbf{V}_2 = -\frac{k_2}{\mu}\,\mathrm{grad}\,p_2, \tag{1}$$

where μ is the viscosity of the fluid. The continuity equations for both media have the form

$$\frac{\partial m_1\rho}{\partial t} + \mathrm{div}\,\rho\mathbf{V}_1 + q = 0, \qquad \frac{\partial m_2\rho}{\partial t} + \mathrm{div}\,\rho\mathbf{V}_2 - q = 0, \tag{2}$$

where ρ is the density of the fluid, and q is the intensity of fluid transfer from medium 2 into medium 1.

It is natural to make the assumption that q is expressed by the relation

$$q = \rho_0\alpha(p_2 - p_1), \tag{3}$$

where ρ_0 is the initial density of the fluid, and α is a certain new characteristic of fractured porous rock, analogous to the heat-transfer coefficient in the theory of heat exchange and characterizing the intensity of fluid exchange between the media. Obviously, the quantity α is proportional to the specific surface area of the fractures. We note that a dependence analogous to (3) for the integral estimate of transfer was adopted in work [(4)].

Assuming that the fluid and both porous media are slightly compressible and substituting relations (1) and (3) into equations (2), we obtain a system of equations describing unsteady filtration of a homogeneous fluid in a fractured porous medium:

$$\begin{aligned} \frac{k_1}{\mu}\Delta p_1 &= (\beta_{c1}+m_1\beta)\frac{\partial p_1}{\partial t} - \alpha(p_2-p_1), \\ \frac{k_2}{\mu}\Delta p_2 &= (\beta_{c2}+m_2\beta)\frac{\partial p_2}{\partial t} + \alpha(p_2-p_1), \end{aligned} \tag{4}$$

where β_{c1}, β_{c2}, and β are, respectively, the compressibility coefficients of media 1, 2, and of the fluid*, and Δ is the symbol of the Laplace operator.

Equations (4) may be written in the form

$$\mathcal{L}_1 p_1 = -\alpha p_2, \qquad \mathcal{L}_2 p_2 = -\alpha p_1, \tag{5}$$

* The coefficient β_{c1} may be defined as the compressibility coefficient of the fractured rock under consideration, assuming the blocks to be incompressible; the coefficient β_{c2}, as the compressibility coefficient of the blocks.

where the linear operators $\mathcal{L}_1$ and $\mathcal{L}_2$ are defined by the relation (I is the identity operator)

$$\mathcal{L}_i = \frac{k_i}{\mu}\Delta - \alpha I - (\beta_{ci}+m_i\beta)\frac{\partial}{\partial t} \quad (i=1,2). \tag{6}$$

Eliminating p_1 and p_2 in turn from (5), we obtain equations in which p_1 and p_2 enter separately:

$$\mathcal{L}p_1 = \alpha^2 p_1, \qquad \mathcal{L}p_2 = \alpha^2 p_2, \tag{7}$$

where the operator $\mathcal{L}$ is defined by the relation

$$\mathcal{L} = \mathcal{L}_1\mathcal{L}_2 = \mathcal{L}_2\mathcal{L}_1 =$$

$$= \frac{k_1 k_2}{\mu^2}\Delta^2 + \alpha^2 I + (\beta_{c1} + m_1\beta)(\beta_{c2} + m_2\beta)\frac{\partial^2}{\partial t^2} - \frac{\alpha(k_1 + k_2)}{\mu}\Delta - \tag{8}$$

$$-\frac{1}{\mu}\left[k_2(\beta_{c1} + m_1\beta) + k_1(\beta_{c2} + m_2\beta)\right]\frac{\partial\Delta}{\partial t} + \alpha\left[(\beta_{c1} + m_1\beta) + (\beta_{c2} + m_2\beta)\right]\frac{\partial}{\partial t}.$$

3. Up to this point both porous media have entered our consideration in a completely symmetric way. However, as already noted above, in a large number of cases simplifications can be introduced, connected with the fact that the permeability of medium 2 is much smaller than the permeability of medium 1, while the porosity and compressibility of medium 2 are much greater than the porosity and compressibility of medium 1. In this case some terms of the basic system (4) turn out to be negligibly small, and this system takes the form

$$\frac{k_1}{\mu}\Delta p_1 + \alpha(p_2 - p_1) = 0,$$

$$(\beta_{c2} + m_2\beta)\frac{\partial p_2}{\partial t} + \alpha(p_2 - p_1) = 0. \tag{9}$$

Eliminating the pressure p_2 from this system, we obtain one third-order equation for p_1—the pressure in the fractures:

$$\frac{\partial p_1}{\partial t} - \frac{k_1}{\alpha\mu}\frac{\partial\Delta p_1}{\partial t} = \frac{k_1}{\mu(\beta_{c2} + m_2\beta)}\Delta p_1. \tag{10}$$

It is interesting to note that the piezoconductivity coefficient entering equation (10) is, as it were, mixed: it corresponds to the permeability of medium 1 and to the porosity and compressibility of medium 2.

In particular, for one-dimensional motion of a liquid by plane waves, equation (10) takes the form

$$\frac{\partial p_1}{\partial t} - \frac{k_1}{\alpha\mu}\frac{\partial^3 p_1}{\partial x^2\partial t} = \frac{k_1}{\mu(\beta_{c2} + m_2\beta)}\frac{\partial^2 p_1}{\partial x^2}, \tag{11}$$

and for one-dimensional axisymmetric motion it has the form

$$\frac{\partial p_1}{\partial t} - \frac{k_1}{\alpha\mu}\frac{\partial}{\partial t}\left[\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial p_1}{\partial r}\right] = \frac{k_1}{\mu\left[\beta_{c2} + m_2\beta\right]}\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial p_1}{\partial r}. \tag{12}$$

4. Let us dwell briefly on the features of the boundary and initial conditions for the problems under consideration. When using system (4), one may proceed from the fact that at the boundary between the free liquid and the porous medium, owing to rapid filtration equalization, equality of the pressures in both media takes place, i.e. $p_1 = p_2$. Therefore in these cases one may prescribe a common pressure at the boundaries of the reservoir, or the total flow rate of liquid through the boundary, as well as the initial distribution of both pressures. When using

when using the simplified system (9), or, what is the same, equation (10), boundary and initial conditions may be prescribed only for the pressure p_1; the pressure p_2 cannot be prescribed arbitrarily, but is determined from the equations of motion. Physically this is connected with the properties of medium 2, in which no equalization occurs because of its low permeability.

5. It should be emphasized that the specific character of rock fracturing manifests itself precisely in nonstationary filtration processes, in which the exchange of liquid between the two constituent porous media has a substantial effect. In stationary processes the pressures in both media become equal, and flow from one medium into the other does not occur on average. Therefore, in order to determine the new characteristic of fractured rock introduced in the present work, α, it is necessary to carry out experiments on essentially nonstationary motion of liquid in the rocks under study, for example, on pressure recovery in shut-in wells.

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