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Abstract

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PHYSICS

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ON QUARTET CORRELATIONS IN LIGHT NUCLEI

(Presented by Academician N. N. Bogolyubov, 18 XI 1959)

It is known that in heavy nuclei pair correlations are observed both in neutron and in proton shells. In light nuclei, however, alongside proton–proton and neutron–neutron correlations, there are also proton–neutron correlations. Pair correlations of nucleons in light nuclei, as shown in ([1]), explain certain regularities in binding energies. However, to explain a number of their basic properties it is not sufficient to take into account only pair correlations. In light nuclei there appear very clearly quartet correlations of nucleons of the type of an α-particle, which explain a number of their important properties; moreover, when an excess of neutrons over protons appears in stable nuclei, i.e., when the mass number A becomes greater than 40, these quartet correlations disappear.

In the present work we consider interactions leading to the formation of quartet correlations of nucleons of the α-particle type. By taking into account pair and quartet correlations, we explain certain regularities in the binding energies of the last neutrons in light nuclei. It is shown that quartet correlations disappear when the number of neutrons exceeds the number of protons, which explains the absence of α-particle properties in nuclei with mass number A greater than 40.

We shall investigate the possibility of the appearance of quartet correlations and study their properties on the basis of the shell model of the nucleus. Let us consider the residual interactions, after subtraction of the self-consistent field, of nucleons located in the outer shell of a light nucleus (say, outside the O^{16} core). We shall characterize the state of a nucleon by a set of quantum numbers s, by the absolute value of the quantum number of the projection of the total angular momentum on the axis of symmetry of the nucleus m, and by the quantity $\sigma = \pm 1$, characterizing the sign of m; moreover, in the case of LS-coupling σ will also determine the spin direction.

Nucleons that are in the nucleus in an S-state relative to one another interact strongly with one another, which leads to the formation of both pair and quartet correlations. This gives us the possibility of taking as the interaction Hamiltonian one that takes into account only the interaction of nucleons with identical quantum numbers s, m. Let us assume that quartet correlations of nucleons arise as a result of the interaction of pairs of nucleons. It should be noted that this assumption is sufficiently well justified in the case when the binding energy within a pair is much greater than the binding energy between pairs. However, we are forced to use a Hamiltonian describing the interaction of pairs, since very great difficulties arise when working with the Hamiltonian of nucleon interaction. In the case of isotopic invariance, different pairs may be formed: either a pair is formed by a neutron and a proton in the states $(s, \sigma m)$, or a pair is formed by two protons (neutrons) in the states (s, m) and $(s, -m)$. For definiteness, we shall take $b_{\rho m}(s)$ to be the operator of a pair of identical nucleons in the states (s, m), $(s, -m)$, with the quantity $\rho = \pm 1$, where $\rho = 1$ corresponds to a proton pair and $\rho = -1$ to a neutron pair.

The model Hamiltonian describing the interactions between pairs, in the case of isotopic invariance, is written in the form

$$H = \sum_{s,m} \{E(s,m) - \lambda\}\{b_m(s)^+ b_m(s) + b_{-m}(s)^+ b_{-m}(s)\} +$$

$$+ \sum_{\substack{s,s',m,m' \\ m \neq m'}} J(s,m;s',m')\, b_m(s)^+ b_{-m}(s)^+ b_{-m'}(s') b_{m'}(s'), \tag{1}$$

where λ is a parameter playing the role of the chemical potential. The operators $b_m(s)$, $b_m(s)^+$ obey the following commutation relations:

$$b_m(s)^+ b_m(s) + b_m(s) b_m(s)^+ = 1;$$

$$\left.\begin{aligned} b_m(s)^+ b_{m'}(s') - b_{m'}(s') b_m(s)^+ = 0, \\ b_m(s) b_{m'}(s') - b_{m'}(s') b_m(s) = 0, \end{aligned}\right\} \quad m \neq m' \text{ or } s = s'. \tag{2}$$

Using the method described in ([2]), we introduce the auxiliary Hamiltonian

$$H' = \sum_{s,m} \{E(s,m) - \lambda\}\{b_m(s)^+ b_m(s) + b_{-m}(s)^+ b_{-m}(s)\} +$$

$$+ \sum_{\substack{s,s',m,m' \\ m \neq m'}} J(s,m;s',m')\{B^*(s,m) b_{-m'}(s') b_{m'}(s') +$$

$$+B(s',m')b_m(s)^+b_{-m}(s)^+ - B^*(s,m)B(s'm')\}. \tag{3}$$

The functions $B(s,m)$ are determined from the condition that $\langle H'\rangle$ be a minimum, so that the mean value $\langle H\rangle$ of the operator H in the eigenstate of the operator H' coincides with $\langle H'\rangle$; as a result we obtain

$$B(s',m') = \langle b_{-m'}(s')b_{m'}(s')\rangle. \tag{4}$$

We carry out the transformation

$$b_m(s) = \{1 - 2a_{-m}(s)^+a_{-m}(s)\}a_m(s), \quad b_{-m}(s) = a_{-m}(s); \tag{5}$$

we note that $a_m(s)^+$, $a_m(s)$ in states with identical quantum numbers s, m behave as Fermi amplitudes.

We shall investigate the possibility of the appearance of quartet correlations by means of the variational principle proposed by N. N. Bogolyubov ([3]), working with the auxiliary Hamiltonian H'. We make the transformation

$$\begin{aligned} a_m(s) &= u_m(s)[1-2\nu_m(s)]\beta_m(s) + v_m(s)\beta_{-m}(s)^+, \\ a_{-m}(s) &= u_m(s)\beta_{-m}(s) - v_m(s)[1-2\nu_m(s)]\beta_m(s)^+, \end{aligned} \tag{6}$$

where $\nu_m(s) = \beta_{-m}(s)^+\beta_{-m}(s)$, $u_m(s)^2 + v_m(s)^2 = 1$. The new quasipair operators $\beta_{\rho m}(s)^+$, $\beta_{\rho m}(s)$ satisfy the commutation relations (2). We define the new vacuum state by $\beta_{\rho m}(s)\Psi = 0$ and find the mean value of the operator H' in this state. Determining $u_m(s)$, $v_m(s)$ from the condition that $\langle H'\rangle$ be a minimum, we obtain the equation

$$\{E(s,m) - \lambda\}u_m(s)v_m(s) + \frac{u_m(s)^2 - v_m(s)^2}{2}\sum_{s',m'} J(s,m;s',m')u_{m'}(s')v_{m'}(s') = 0, \tag{7}$$

which has the same form as the equations in ([4]) that take pair correlations into account.

Both the asymptotic solution of (7) for $J \to 0$ and the solution of (7) for $J = \text{const}$, $\rho = \text{const}$ lead to the conclusion that the formation of quartet cor-

correlations is energetically favorable. The energy of the ground state with correlated quartets and pairs lies below the energy of a state in which there are only pair correlations. The first excited state, associated with the breakup of a quartet, is separated from the ground state by an energy gap.

It should be noted that equation (7) can be obtained not only as set forth above, by means of the variational principle, but also by the correlation-splitting method proposed by N. N. Bogolyubov (5).

Let us consider the influence of quartet correlations, along with pair correlations of nucleons, on, for example, the binding energy of the last neutron in light nuclei. If $\langle H' \rangle$ determines the ground-state energy with an even number of pairs, then the ground-state energy of a nucleus with an odd number of pairs has the form $\langle \beta_{m_0}(s) H' \beta_{m_0}(s_0)^+ \rangle$. To find the binding energy of the last nucleon in a nucleus it is necessary to calculate

$$\langle \beta_{m_0}(s_0) H' \beta_{m_0}(s_0)^+ \rangle_{N=2n_0 \pm 1} - \langle H' \rangle_{N=2n_0}.$$

In the approximation (6), $J = \text{const}$, $\rho = \text{const}$, we obtain

$$\begin{aligned} \langle \beta_{m_0}(s_0) H \beta_{m_0}(s_0)^+ \rangle_{2n_0+1} - \langle H' \rangle_{2n_0} &= -\frac{\Omega+1}{2\rho} - \frac{e^{2/G}+1}{e^{2/G}-1} \frac{2n_0 - \Omega + 1}{2\rho}, \\ \langle \beta_{m_0}(s_0) H' \beta_{m_0}(s_0)^+ \rangle_{2n_0-1} - \langle H' \rangle_{2n_0} &= -\frac{\Omega+1}{2\rho} + \frac{e^{2/G}+1}{e^{2/G}-1} \frac{2n_0 + \Omega - 1}{2\rho}, \end{aligned} \tag{8}$$

where ρ is the level density, Ω the number of levels, and $G = -J\rho$. Expressions taking into account pair correlations of nucleons have a similar form. From (8) it is evident that much more energy is needed to break up a quartet than to remove an uncorrelated pair, just as the breakup of a pair is associated with a larger energy than the removal of an uncorrelated nucleon from a nucleus.

Indeed, in those nuclei in which there are neutrons in the outer shell that are bound neither in quartets nor in pairs, such as ${}_{11}\mathrm{Na}_{13}^{24}$, ${}_{12}\mathrm{Mg}_{13}^{25}$, ${}_{18}\mathrm{Al}_{15}^{28}$, ${}_{15}\mathrm{P}_{17}^{32}$, ${}_{16}\mathrm{S}_{19}^{35}$, ${}_{17}\mathrm{Cl}_{19}^{36}$, ${}_{18}\mathrm{Ar}_{23}^{41}$, ${}_{19}\mathrm{K}_{21}^{40}$, the binding energy of the last neutron is of the order of 7-8 MeV. In the nuclei ${}_{11}\mathrm{Na}_{12}^{23}$, ${}_{12}\mathrm{Mg}_{14}^{26}$, ${}_{13}\mathrm{Al}_{14}^{27}$, ${}_{15}\mathrm{P}_{16}^{31}$, ${}_{16}\mathrm{S}_{18}^{34}$, ${}_{17}\mathrm{Cl}_{18}^{35}$, ${}_{18}\mathrm{Ar}_{20}^{38}$, two outer neutrons are bound in a pair, and additional energy is required to break it up (4), which is confirmed by the binding energy of the neutron in these nuclei, equal to 11-13 MeV. Correlation of an outer proton and neutron situated in identical energy states is observed in ${}_{11}\mathrm{Na}_{11}^{22}$, ${}_{13}\mathrm{Al}_{13}^{26}$, ${}_{15}\mathrm{P}_{15}^{30}$, ${}_{17}\mathrm{Cl}_{17}^{34}$, ${}_{19}\mathrm{K}_{19}^{38}$; the binding energy of the last nucleon in these nuclei is of the order of 11-12 MeV. In nuclei of the α-particle type, ${}_{10}\mathrm{Ne}_{10}^{20}$, ${}_{12}\mathrm{Mg}_{12}^{24}$, ${}_{14}\mathrm{Si}_{14}^{28}$, ${}_{16}\mathrm{S}_{16}^{32}$, ${}_{18}\mathrm{Ar}_{18}^{36}$, and ${}_{20}\mathrm{Ca}_{20}^{40}$, quartet correlations of neutrons and protons in states with identical quantum numbers s, m are observed; the binding energy of the last neutron in them is of the order of 15-17 MeV.

From what has been set forth above it is clear that the data on the binding energy of the last neutron in nuclei confirm our conclusions concerning the existence in light nuclei of pair and quartet correlations.

Thus, some regularities in the region of light nuclei, understandable from the point of view of the α-particle model of the nucleus, are explained in the present work within the framework of the shell model by taking quartet correlations into account.

When an excess of the number of neutrons over the number of protons appears, the magnitude of the effective interaction between pairs must decrease strongly in comparison with $J(s, m; s', m')$ in (1), which refers to the case of strict isotopic invariance. It is known that the main contribution to $J(s, m; s', m')$ is made by the resonance interaction of nucleons with equal energies; when an excess of the number of neutrons over the number of protons appears, this resonance is strongly smeared out, which will lead to a sharp weakening of the interaction between

pairs. Further, the formation of quartet correlations when the number of neutrons exceeds the number of protons becomes energetically unfavorable because of the separation of the Fermi surfaces for neutrons and protons. Both of these reasons lead to a sharp disappearance of α-particle properties in nuclei with $A > 40$, where for all stable nuclei $N > Z$.

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References

[1] V. B. Belyaev, B. N. Zakharyev, V. G. Solov' ev, ZhETF, 38, No. 3 (1960).
[2] N. N. Bogolyubov, D. N. Zubarev, Yu. A. Tserkovnikov, DAN, 117, 788 (1957).
[3] N. N. Bogolyubov, DAN, 119, 244 (1958).
[4] V. G. Solov' ev, ZhETF, 35, 823 (1958); Nucl. Phys., 9, 655 (1959).
[5] N. N. Bogolyubov, Usp. fiz. nauk, 67, 549 (1959).
[6] V. G. Solov' ev, ZhETF, 36, 1869 (1959).

Note: Figure translations are in progress. See original paper for figures.