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1960

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Abstract

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HYDROMECHANICS

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A CONVERGING WAVE IN A PLASTIC MEDIUM

(Presented by Academician N. N. Semenov on XI 13, 1959)

The problem of an explosion in a plastic compressible medium was studied by A. S. Kompaneets in [1]. In that work a simplified law of compression was adopted, which made it possible to obtain simple formulas for the law of motion of the front of a diverging spherical wave.

Using an analogous law of compression (we shall assume that, at any pressure different from zero, the density of the medium reaches the limiting value $\rho_* > \rho_0$), we shall consider the problem of a converging plastic wave, which may be formulated as follows.

Let, at the time $t = \tau_0$, on the free boundary of a spherical layer with initial outer radius a_0, a pressure arise that varies according to a prescribed law $P = P_0 F(t/\tau_0)$ (in the more general case one may take $F = F(t/\tau_0, x)$. The pressure on the inner surface of the spherical layer with initial radius b_0 is equal to zero. At the moment the pressure is applied, a shock wave begins to propagate through the material.

We shall assume that behind the front the medium is incompressible, $\rho_\phi = \rho_* > \rho_0$, and that the plasticity condition is satisfied

$$\sigma_r - \sigma_\theta = k + m(\sigma_r + 2\sigma_\theta), \tag{1}$$

in which k and m are assumed to be known constants, and σ_r and $\sigma_\theta = \sigma_\phi$ are the principal stresses. Values of quantities at the wave front are denoted by the subscript ϕ. We shall also suppose that the initial density of the material in the spherical layer depends on the radius, $\rho_0 = \rho_0(r_0)$.

We shall solve the problem in Lagrangian variables, in which the equations of continuity and motion are written as follows:

$$\frac{\partial r}{\partial r_0} = \frac{r_0^2 \rho_0(r_0)}{r^2 \rho}, \tag{2}$$

$$\rho \frac{\partial u}{\partial t} = \frac{\partial r}{\partial r_0} \frac{\partial \sigma_r}{\partial r_0} - \frac{2(\sigma_r - \sigma_\theta)}{r}. \tag{3}$$

Here $u = \partial r/\partial t = \dot{r}$; t is time; r and r_0 are the current and initial coordinates of a particle.

Using (1) and (2), equation (3) can be reduced to the form

$$\frac{\partial}{\partial r_0}\left[r^\alpha \left(\frac{k}{3m} - P\right)\right] = \rho_0(r_0) r_0^2 r^{\alpha-2} \frac{\partial u}{\partial t}, \tag{4}$$

$$\alpha = -\frac{6m}{2m+1} \leqslant 0, \qquad P = -\sigma_r.$$

The boundary conditions for equations (2) and (4) are the conservation laws at the wave front, equality of pressures on the free surface $r_0 = a_0$, and the condition of continuity of the current radius,

$$P_\phi = \rho_0(R)\varepsilon_\phi \dot{R}^2, \qquad u_\phi = \varepsilon_\phi \dot{R}, \qquad r_\phi = R = r_0,$$

$$P(a_0, t) = P_0 F(t/\tau_0), \qquad \varepsilon_\phi = 1 - \rho_0/\rho_*. \tag{5}$$

Introduce the dimensionless quantities

$$a = a_*/a_0, \qquad b = b_*/a_0, \qquad x = R/a_0, \qquad s = r_0/a_0, \qquad \bar{r} = r/a_0,$$

$$\bar{\rho} = \rho/\rho_*, \qquad \varkappa = k/3mP_0, \qquad \bar{k} = k/P_0, \qquad \tau = t/\tau_0, \qquad \bar{u} = \partial\bar{r}/\partial\tau, \tag{6}$$

$$dx/d\tau = -\sqrt{y}, \qquad y = \rho_* \dot{R}^2/P_0, \qquad \tau_0 = a_0\sqrt{\rho_*/P_0},$$

where a_* and b_* are the outer and inner radii of the spherical layer at the instant of time t.

Integrating (2) with allowance for conditions (5), we find

$$\bar{r}^3 = x^3 + 3\int_x^s \bar{\rho}_0(s) s^2 ds, \qquad a^3 = x^3 + 3\int_x^1 \bar{\rho}_0 s^2 ds,$$

$$\bar{u} = -\frac{\lambda(x)}{\bar{r}^2}, \qquad \lambda = \varepsilon_\phi x^2 \sqrt{y(x)}, \qquad \varepsilon_\phi = 1 - \bar{\rho}_0(x). \tag{7}$$

Integrating equation (4) with allowance for (7), we obtain

$$\bar{r}^\alpha \bar{p} = \varkappa(\bar{r}^\alpha - x^\alpha) + x^\alpha \bar{P}_\phi - \frac{d\lambda}{dx}\sqrt{y}\int_x^s \bar{\rho}_0(s)\bar{r}^{\alpha-4}s^2 ds + 2\lambda^2 \int_x^s \bar{\rho}_0 \bar{r}^{-\alpha-7}s^2 ds,$$

$$\bar{P}_\phi = \bar{\rho}_0(x)\varepsilon_\phi y, \qquad \alpha \neq 0. \tag{8}$$

If the law of motion of the wave front is known, then by formulas (7) and (8) one can find the distribution of pressure and velocity throughout the region $x \le s \le 1$.

The ordinary differential equation for the velocity of the wave front is obtained from (8), if the condition on the free surface is used. Assuming also that $\rho_0 = \rho_1 s^n$, when all the integrals in equalities (7) and (8) are evaluated, we finally find

$$\frac{dz}{dx} = K(x)z + N(x,\tau), \qquad \frac{d\tau}{dx} = -\frac{x^2\varepsilon(x)}{\sqrt{\varepsilon_1 z}}, \qquad \tau = 1, \quad x = z = 1. \tag{9}$$

Here

$$z = \frac{yx^4\varepsilon^2}{\varepsilon_1}, \qquad \varepsilon = 1 - \beta x^n, \qquad \varepsilon_1 = 1 - \beta, \qquad \rho_0(a_0) = \beta\rho_*,$$

$$K = \frac{4(\alpha-1)x^2\varepsilon\,(a^{\alpha-4} - x^{\alpha-4})}{(\alpha-4)(a^{\alpha-1} - x^{\alpha-1})} + \frac{2\beta(\alpha-1)x^{n+\alpha-2}}{a^{\alpha-1} - x^{\alpha-1}},$$

$$a = \left[x^3 + \frac{3\beta(1 - x^{n+3})}{n+3}\right]^{1/3},$$

$$N = \frac{2\beta(\alpha-1)x^2\varepsilon\,[\delta_1 - a^\alpha F(\tau,x)]}{\varepsilon_*(a^{\alpha-1} - x^{\alpha-1})}, \qquad \delta_1 = \varkappa(a^\alpha - x^\alpha).$$

For integrating equation (9) it is necessary to know its asymptotics as $x \to 0$ and the value of the derivative dz/dx at the point $x = 1$.

After a number of calculations we find

$$3\left(\frac{dz}{dx}\right)_1 = 4\varepsilon_1 + 8 - 2\sqrt{\varepsilon_1}\left(\frac{dF}{d\tau}\right)_1 + \frac{2}{\varepsilon_1\beta}\left(\frac{d\varepsilon}{dx}\right)_1 + \delta_2, \qquad \delta_2 = 2\alpha\beta(\varkappa - 1); \tag{10}$$

$$\lim_{x\to 0}\tau = A, \qquad z = B_1 x^{4(\alpha-1)/(\alpha-4)} + \cdots \qquad (x \to 0,\ n > 0); \tag{11}$$

Fig. 1

Figure 1: Fig. 1

Fig. 2

Figure 2: Fig. 2

$$z = B_2 x^{\omega} + \cdots, \qquad \omega = \frac{2(\alpha - 1)[2\varepsilon_1 - \beta(\alpha - 4)]}{\alpha - 4} \qquad (n = 0,\ \alpha \leq 0). \quad (12)$$

However, if the external pressure falls very rapidly, the wave may stop before reaching the center.

Equations (8) and (9) are valid in the case $\alpha \neq 0$; but if $m = 0$, then the plasticity condition is transformed to the form $\sigma_r - \sigma_\theta = k$, and the solution is given by the same equations, in which one must set

$$\alpha = 0, \qquad \delta_1 = 2\bar{k}\ln(x/a), \qquad \delta_2 = -4\beta\bar{k}.$$

Let us also note that equations (9) also describe the solution of the problem of the collapse of a spherical layer of incompressible plastic material. In this case one must set

$$\bar{r}^3 = s^3 + x^3 - b_0^3, \qquad x = b, \qquad \bar{P}_\phi = 0, \qquad z = yx^4,$$

$$k = \frac{4(\alpha - 1)x^2\left(a^{\alpha-4} - x^{\alpha-4}\right)}{(\alpha - 4)\left(a^{\alpha-1} - x^{\alpha-1}\right)}, \qquad N = \frac{2(\alpha - 1)x^2\left[\delta_1 - a^{\alpha}F\right]}{a^{\alpha-1} - x^{\alpha-1}}. \quad (13)$$

The asymptotic solution in these cases as well is given by formula (11). The boundary conditions for this problem will be, for $\tau = 1$, $x = b_0$, $y = 0$ or $y = y_0$, when the motion of the liquid already exists at the initial moment. The latter case can be realized, for example, if the spherical layer of the compacted material had been compressed by an external pressure to the limiting state $\rho_\phi = \rho_*$; then, after the disappearance of the shock wave (at the boundary with the cavity), the compressed material continues to collapse as incompressible.

Fig. 1

Fig. 2

Figures 1 and 2 show the results of numerical integration of equations (8). Curve 1 corresponds to the solution of the problem for the parameter values: $F = t^{-1,2}$, $\varepsilon_* = 0,1$, $n = 0$, $\chi = -0,1$, $\alpha = -1$; curve 2 for $F = t^{-3}$, $\varepsilon_* = 0,1$, $n = 0$, $k = -0,1$, $\alpha = 0$; curve 3 for $F = 1$, $\varepsilon_* = 0,2$, $n = 1$, $\beta = 0,8$, $\chi = -0,01$,

$\alpha = -1$. (In the case of constant external pressure, or $F = F(x)$, equations (8) are integrated in quadratures.)

From the numerical calculation one can determine the values of the coefficients A and B in the asymptotics (11) and (12). In case 2, $A \approx 0,44$, $B \approx 1,195$, $\omega = 1,9$; in case 3, $A \approx 0,166$; $B \approx 1,464$; $\omega = 1,6$.

Knowing the behavior of the solution as $x \to 0$, one can investigate the concentration of energy at the center. The energy per unit volume is composed of the kinetic energy, the energy dissipated at the wave front, and the energy dissipated due to the work of the forces of plastic deformation; therefore

$$\int_a^1 a^2 F\, da = \frac{1}{2}\int_x^1 \bar{\rho}_0(s)\, \bar{u}^2 s^2\, ds + \frac{1}{2}\int_x^1 \varepsilon_\phi P_\phi s^2\, ds + 2\int_1^\tau \left[\int_x^1 \frac{(\bar{\sigma}_r - \bar{\sigma}_\theta)(1-\varepsilon_\phi)\bar{u}s^2\, ds}{\bar{r}}\right] d\tau. \tag{14}$$

Studying the behavior of these integrals as $x \to 0$ and $s \to 0$, one can establish that a finite concentration of energy at the center occurs only in the case

the collapse of a spherical layer of an incompressible fluid (the second integral in (14) is absent) under the condition $n = mk = 0$ ($\bar{k}$ arbitrary). In all other cases all the integrals (14) converge, which means a decrease in the cumulation of energy at the center.

Analogously to what was set forth above, the problem can be solved under the assumption of a variable compaction of the substance depending on the pressure amplitude at the wave front [2]. However, just as in [3,4], in this case as well one would have to solve an integro-differential equation for the velocity of the shock-wave front.

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Note: Figure translations are in progress. See original paper for figures.