V. I. SKOBELKIN

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Abstract

Full Text

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VARIATIONAL PRINCIPLES FOR DETERMINING THE BASIC CHARACTERISTICS OF A FERROMAGNET ON THE BASIS OF CALCULATING THE HYSTERESIS LOOP

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As shown in ([1]), the magnetization of a ferromagnet in the field of currents satisfies the variational equation

$$\delta \int_{\omega} (w + u)\, d\omega = 0,$$

where

$$w = \frac{1}{4\pi} \int_0^{\mathbf{B}} \mathbf{H}\, d\mathbf{B}$$

is the density of the magnetic energy of the field;

$$u = -\frac{1}{2c} \mathbf{j}(\psi \nabla \vartheta - \vartheta \nabla \psi)$$

is the density of the potential function of the currents; $\mathbf{j}$ is the current density; ψ and ϑ are functions of the magnetic flux, which are related to the induction $\mathbf{B}$ by the relation $\mathbf{B} = \nabla\psi \times \nabla\vartheta$. The functional $E = \int_\omega L\, d\omega$, where $L = w + u$ is the Lagrangian of the field, assumes the least value for the actual magnetic field.

In the case of irreversible magnetization, the entropy S of the ferromagnet changes; therefore, in order to describe the process of magnetization, it is necessary to invoke the second law of thermodynamics. The presence of magnetostrictive phenomena in a ferromagnet changes the surface σ of the ferromagnet; consequently, instead of the principal problem considered in ([1]), the variational problem in this case becomes nonprincipal (with movable boundaries). As the field-determining quantities we choose the Cartesian coordinates Y, Z, and as independent variables x, ψ, ϑ ([2,3]).

The variational equation for the field will then take the form

$$\Delta(Y, Z) = \delta E^* + \int_\omega \Pi \delta S \, d\omega - \int_l L^* \frac{D(Y, Z)}{D(\psi, \vartheta)} \delta(\psi\vartheta) \, dx = 0, \tag{1}$$

where l is the contour of the surface σ; $\Pi = -\partial L^*/\partial S$; L^* is the generalized Lagrangian of the field, taking into account the change of the boundary of the surface σ of the ferromagnet:

$$L^* = w + u - \int_0^{\mathbf{B}(\sigma)} \mathbf{B} \, d\mathbf{H}.$$

The last term is equal to the magnetic pressure on the surface σ of the ferromagnet. In equation (1), the variation δE^* is the complete variation with allowance for the change in entropy and the flux of magnetic induction at the boundary surface σ. From equation (1) one obtains the usual Maxwell equation ([1])

$$\operatorname{rot} \mathbf{H} = \frac{4\pi}{c} \mathbf{j}$$

and the boundary condition

$$[H_\tau]_\sigma = 0.$$

The equality of the components of the magnetic induction $\mathbf{B}$ normal to the surface σ follows from the continuity of the functions ψ, ϑ. Equation (1), equivalent to Maxwell' s equation, is satisfied for any surface σ, independently of its change in the magnetization process.

To take into account the change of the surface σ itself, an additional condition is necessary, one that follows from the second law of thermodynamics. If this condition is introduced into equation (1), then from it, in addition to Maxwell' s equations, one obtains a new, supplementary equation determining the form of the surface σ. Since the ferromagnet is an adiabatically isolated system, the variation of the entropy entering into (1) must vanish for the actual state. Indeed, if one considers infinitely small deviations from the actual state, then for these deviations, up to infinitely small quantities of higher order, equation (1) is also satisfied exactly. Therefore, if the field equation (1) is regarded as a certain nonholonomic constraint, then this constraint is always satisfied for infinitely small deviations. Consequently, infinitely small deviations from the actual state may be regarded as possible nonequilibrium states of the ferromagnet. If in equation (1) $\delta S \neq 0$, then such deviations would be possible for which $\delta S > 0$, and then the system, being adiabatically isolated, would pass into a new state with a larger value of S, which is impossible, since the solution of equation (1) corresponds to thermodynamic equilibrium.

Putting $\delta S = 0$ in (1), we obtain

$$\Delta(Y,Z) = \delta E^* - \int_l L^* \frac{D(Y,Z)}{D(\psi,\vartheta)}\,\delta(\psi\vartheta)\,dx = 0. \tag{2}$$

It follows directly from (2) that $\delta\Phi^+ = 0$, i.e., the positive magnetic flux Φ^+ through the surface σ has a maximum. The positive flux Φ^+ is determined by the condition

$$\Phi^+ = \int_\sigma \mathbf{Bn}\varepsilon\,d\sigma,$$

where $\varepsilon = 1$ if $\mathbf{Bn} \geq 0$, and $\varepsilon = 0$ if $\mathbf{Bn} < 0$.

In the general case the thermodynamic potential $\bar{\varphi}$ of a unit volume of the ferromagnet ([4]) will be:

$$d\bar{\varphi} = -S\,dT - u_{ik}d\sigma_{ik} - \frac{1}{4\pi}\mathbf{B}\,d\mathbf{H},$$

whence

$$\bar{\varphi} = \varphi_0(M) + \mathcal{E}_{\text{an}} - \mathbf{HM} - \frac{H^2}{8\pi} + \mathcal{E}_{\text{mu}} - \mathcal{E}_{\text{el}}. \tag{3}$$

In (3) $\mathbf{H}$ is the magnetic-field intensity inside the ferromagnet; $\mathbf{M}$ is the magnetization of a unit volume; $\mathcal{E}_{\text{an}}$ is the anisotropy energy

$$\left(\mathcal{E}_{\text{an}} = \frac{\beta_{ik}}{2} M_{iM}k\right);$$

β_{ik} is a dimensionless symmetric tensor of rank 2, whose components are functions of the temperature T; $\mathcal{E}_{\text{mu}}$ is the magnetoelastic energy, equal to $-\lambda_{iklm}\sigma_{ik}M_{lM}m$, where λ_{iklm} is a tensor of rank 4, symmetric with respect to the pairs of indices i,k and l,m (but not with respect to interchange of the pair i,k with the pair l,m ([4])); $\mathcal{E}_{\text{el}}$ is the ordinary elastic energy; $\varphi_0(M)$ is the thermodynamic potential of a unit volume of the substance at $\mathbf{H} = 0$, regarded as a function of the independent variable $\mathbf{M}$ along with the other thermodynamic variables. If only exchange interactions are taken into account and relativistic interactions are neglected, then φ_0 depends only on the absolute value M (the order of magnitude of the ratio of relativistic interactions to the exchange interaction is characterized by the ratio $\nu = \mathcal{E}_{\text{an}}/N\theta_C$, where N is the number of atoms in 1 cm^3; θ_C is the Curie-point temperature; $\nu \sim 10^{-4} \div 10^{-6}$ ([4])).

For uniaxial crystals

$$\mathcal{E}_{\text{an}} = \frac{\beta}{2}\left(M_x^2 + M_y^2\right) = \frac{\beta}{2} M^2 \sin^2 \vartheta,$$

where θ is the angle between $\mathbf{M}$ and the z-axis, chosen along the principal axis of symmetry of the crystal; β is the anisotropy constant. For a cubic crystal,

$$\mathcal{E}_{\text{an}} = -\frac{\beta}{2}\left(M_x^4 + M_y^4 + M_z^4\right),$$

$$\mathcal{E}_{\text{el}} = \frac{k_1}{2}\left(\sigma_{xx}^2 + \sigma_{yy}^2 + \sigma_{zz}^2\right) + k_2\left(\sigma_{xx}\sigma_{yy} + \sigma_{xx}\sigma_{zz} + \sigma_{yy}\sigma_{zz}\right) + k_3\left(\sigma_{xy}^2 + \sigma_{xz}^2 + \sigma_{yz}^2\right).$$

We neglect the energy of magnetic inhomogeneity in the transition layers of domains, which is proportional to the magnetization gradients.

The requirement that Φ^+ be a maximum leads to the equation

$$\delta \int_{\sigma^+} (\mathbf{H} + 4\pi\mathbf{M})\, \mathbf{n}\, d\sigma^+ = 0 \tag{4}$$

under the conditions

$$u_{ik} = -\left(\frac{\partial \bar{\varphi}}{\partial \sigma_{ik}}\right)_{T,\mathbf{H}}, \tag{5}$$

where u_{ik} is the strain tensor of the ferromagnet. Let $\mathbf{M} = \mathbf{M}_0 + \vec{\mathfrak{M}}$, where $\mathbf{M}_0$ is the magnetization determined by the initial magnetization curve; $\vec{\mathfrak{M}}$ is the deviation of the magnetization in the reverse process as a result of hysteresis. Then from the equation $\delta S = 0$ it follows either that $dS/d\Phi^+ = 0$ (the initial magnetization curve), or that $\delta\Phi^+ = 0$ (metastable states corresponding to hysteresis). If the direction cosines s_1, s_2, s_3 of the magnetization and the modulus $\mathfrak{M} = |\vec{\mathfrak{M}}|$ are taken as independent variables, then, in order to determine s_i and $\mathfrak{M}$ in the region of the hysteresis loop, we obtain the equation

$$\delta\left[\Phi^+ - \lambda\left(s_1^2 + s_2^2 + s_3^2 - 1\right)\right] = 0, \tag{6}$$

where

$$\Phi^+ = \int_{\sigma^+} \left\{\mathbf{H} + 4\pi\left(\mathbf{M}_0 + \vec{\mathfrak{M}}\right)\right\} \mathbf{n}\, d\sigma;$$

λ is the Lagrange parameter; $s_1^2 + s_2^2 + s_3^2 = 1$.

A qualitative estimate of equation (6) shows the presence of two roots (for a given $\mathbf{M}_0$), which, depending on the change of $\mathbf{H}$, form a closed curve (the hysteresis loop) corresponding to possible metastable states in the region of irreversible magnetization. If in this case $\omega = \text{const}$ and $\delta S = 0$, then, as follows from the first law of thermodynamics and the variational principle (1) (for the region where there are no currents),

$$\delta U = T\delta S + \frac{1}{4\pi}\int_\omega \mathbf{H}\,\delta\mathbf{B}\,d\omega = 0,$$

and therefore the conditions of adiabatic isolation of the system are satisfied.

The mechanism of hysteresis has been investigated rather fully in work [5], especially in the initial region of displacement of domain boundaries. The present theory makes it possible to calculate quantitatively the configuration of the entire hysteresis loop and relates the various characteristics (constants) of the ferromagnet to the shape of the loop.

Moscow State University
named after M. V. Lomonosov

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Note: Figure translations are in progress. See original paper for figures.