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Abstract

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New Kinetic Equations in the Theory of Monatomic Gases

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In the present paper we shall indicate new kinetic equations for the theory of monatomic gases. In contrast to the Boltzmann kinetic equation, our equations are purely integral equations and contain boundary conditions on the surfaces of bodies immersed in the gas. Owing to the inclusion of boundary conditions in the kinetic equations, the latter may be regarded as equations of the kinetics (aerodynamics) of rarefied gases. The Boltzmann kinetic equation is a substantially impoverished consequence of our equations.

In order not to distract attention by secondary details, we shall assume that the external field of mass forces is absent, regard all gas atoms as identical, and take into account only pair collisions of atoms.

I. Let us introduce notation: x_1, x_2, x_3 are Cartesian coordinates; t is time; $\mathbf{r}$ is the radius vector with projections x_1, x_2, x_3; $\mathbf{u}$ is the velocity vector of a gas atom, having projections u_1, u_2, u_3; $d\Omega = dx_1 dx_2 dx_3$ is the spatial volume element adjoining the point with radius vector $\mathbf{r}$; $d\omega = du_1 du_2 du_3$ is the volume element in velocity space adjoining the end of the vector $\mathbf{u}$; $\mathbf{r}_s$ is the radius vector (at the time τ_s) of such a point of the surface s of the immersed body that an atom with velocity $\mathbf{u}$, having flown out of this point at the time τ_s, is found (in the absence of collisions) at the time t at the point with radius vector $\mathbf{r}$ ($\tau_s = -\infty$ if there is no corresponding point on s); $\mathbf{n}$ is the unit vector of the normal to the surface s of the immersed body at the point $\mathbf{r}_s$ at the time τ_s; θ_s is the temperature on the surface of the immersed body at the point $\mathbf{r}_s$ at the time τ_s; σ is the collision cross section of atoms, which, generally speaking, depends on their relative velocity; $\mathbf{v}_s$ is the velocity of the point $\mathbf{r}_s$ of the surface of the moving immersed body at the time τ_s, the motion of the body being regarded as prescribed.

For brevity of notation, we shall agree to write a function $F(\dots, A_1, A_2, A_3, \dots)$ of the projections A_1, A_2, A_3 of the vector $\mathbf{A}$ in the form $F(\dots, \mathbf{A}, \dots)$.

II. Let us introduce definitions:

1) By the distribution function f we shall mean such a function of the arguments $\mathbf{r}, \mathbf{u}$, and t that the quantity

$$dn_1 = f(\mathbf{r}, \mathbf{u}, t)\, d\Omega\, d\omega \tag{1}$$

is the mathematical expectation, at the time t, of the number of atoms in the volume element $d\Omega$ with velocities from the element $d\omega$.

2) By the birth of an atom we shall mean the last collision it experienced, as a result of which it acquired its present velocity (it is clear that every atom was born at some moment of time at some point in space, and that birth may take place within the gas and on the surface of a body immersed in the gas).

3) By the internal birth function Φ we shall mean such a function of the arguments $\mathbf{r}$, $\mathbf{u}$, and t that the quantity

$$dn_2 = \Phi(\mathbf{r}, \mathbf{u}, t)\, d\Omega\, d\omega\, dt \tag{2}$$

is the mathematical expectation of the number of atoms born during the time interval $(t, t + dt)$ in the volume element $d\Omega$ and having velocities from $d\omega$.

4) By the boundary birth function Ψ we shall mean such a function of the arguments $\mathbf{r}$, $\mathbf{u}$, and t that the quantity

$$dn_3 = \Psi(\mathbf{r}, \mathbf{u}, t)\, ds\, d\omega\, dt \tag{3}$$

is the mathematical expectation of the number of atoms born during the time interval $(t, t + dt)$ on the element ds of the surface of the body being flowed around and having, after birth, a velocity from $d\omega$; here the surface element ds is adjacent to the point of the surface of the body being flowed around with radius vector $\mathbf{r}$.

5) By the probability Π of free motion of an atom over the time interval (τ, t) we shall mean the probability of the random event consisting in the fact that an atom, known to possess velocity $\mathbf{u}$ at the time τ and located at that moment at the point $\mathbf{r} - \mathbf{u}(t - \tau)$, experiences no collision during the time interval (τ, t) and at the time t is found at the point with radius vector $\mathbf{r}$ (obviously, Π is a function of the arguments $\mathbf{r}$, $\mathbf{u}$, τ, and t).

6) By the internal impact transform T we shall mean a function of three velocity vectors $\mathbf{u}_1$, $\mathbf{u}_2$, and $\mathbf{u}$ having the property that the quantity

$$dn_4 = T(\mathbf{u}_1, \mathbf{u}_2, \mathbf{u})\, d\omega \tag{4}$$

is the mathematical expectation of the random event consisting in the fact that, as a result of a collision of two atoms with velocities $\mathbf{u}_1$ and $\mathbf{u}_2$ that is known to have occurred, after the collision either the first or the second atom has a velocity in the element $d\omega$ of velocity space*.

7) By the boundary impact transform $\widetilde{T}$ we shall mean a function of two velocity vectors $\mathbf{u}_1$ and $\mathbf{u}$ of atoms, of the normal vector $\mathbf{n}$, temperature θ, and velocity $\mathbf{v}$ of a point of the surface of the body being flowed around, having the property that the quantity

$$dn_5 = \widetilde{T}(\mathbf{u}_1, \mathbf{u}, \mathbf{n}, \mathbf{v}, \theta)\, d\omega \tag{5}$$

is the probability that, as a result of a collision known to have occurred between an atom with velocity $\mathbf{u}_1$ and a point of the surface of the body being flowed around at which $\mathbf{n}$ is the normal, θ the temperature, and $\mathbf{v}$ the velocity, an atom is born with a velocity from $d\omega$.

III. Between the functions f, Π, Φ, and Ψ there exist relations that can be written in the form of the equalities:

$$f(\mathbf{r}, \mathbf{u}, t) = \frac{1}{|(\mathbf{u}-\mathbf{v})_n|}\Psi(\mathbf{r}_s, \mathbf{u}, \tau_s)\Pi(\mathbf{r}, \mathbf{u}, \tau_s, t) + \int_{\tau_s}^{t} \Phi(\mathbf{r}-(t-\tau)\mathbf{u}, \mathbf{u}, \tau)\Pi(\mathbf{r}, \mathbf{u}, \tau, t)\, d\tau; \tag{6}$$

$$\Pi(\mathbf{r}, \mathbf{u}, \tau, t) =$$

$$= \exp\left\{-\int_{\tau}^{t}\left[\int_{-\infty}^{+\infty}\iint |\mathbf{u}-\mathbf{u}_1|\,\sigma(|\mathbf{u}-\mathbf{u}_1|) f(\mathbf{r}-(t-q)\mathbf{u}, \mathbf{u}_1, q)\, d\omega_1\right] dq\right\}; \tag{7}$$

* In representing the transforms T and $\widetilde{T}$, it may be necessary to use Dirac delta functions; this will occur if

$$\Phi(\mathbf{r}, \mathbf{u}, t) = \tag{8}$$

$$= \frac{1}{2}\int_{-\infty}^{+\infty}\iiiint\!\!\iint |\mathbf{u}_2-\mathbf{u}_1|\,\sigma(|\mathbf{u}_2-\mathbf{u}_1|) f(\mathbf{r}, \mathbf{u}_1, t) f(\mathbf{r}, \mathbf{u}_2, t) T(\mathbf{u}_1, \mathbf{u}_2, \mathbf{u})\, d\omega_1\, d\omega_2;$$

$$\Psi(\mathbf{r}_s, \mathbf{u}, t) = \int_{(\mathbf{n}_1-\mathbf{v})_n<0}\!\!\iint f(\mathbf{r}_s, \mathbf{u}_1, t)\, |(\mathbf{u}_1-\mathbf{v})_n|\, \widetilde{T}(\mathbf{u}_1, \mathbf{u}, \mathbf{n}, \mathbf{v}, \theta_s)\, d\omega_1. \tag{9}$$

If the functions σ, T, and $\widetilde{T}$ are known from theoretical considerations or experiment, and $\mathbf{v}$ is specified, then (6), (7), (8), and (9) (with the corresponding boundary conditions for the temperature θ_s) form a system of four integral equations for finding the four functions f, Π, Φ, and Ψ. The structure of these equations is such that the functions Π, Φ, and Ψ can be eliminated from them by direct substitution. As a result of the elimination, one obtains a single integral equation

$$f = V(f), \tag{10}$$

where V is a certain integral operator acting on f.

For the integral equation (10) one can formulate a statement of the problem of the kinetics (aerodynamics) of monatomic gases that covers gas flows from free-molecular flows to flows described by the scheme of hydrodynamics of an ideal fluid.

VI. If both sides of equation (10) are acted upon by the operator

$$\frac{d}{dt} = \frac{\partial}{\partial t} + u_1 \frac{\partial}{\partial x_1} + u_2 \frac{\partial}{\partial x_2} + u_3 \frac{\partial}{\partial x_3}, \tag{11}$$

then from equation (10) one obtains the equation

$$\frac{df}{dt} = \Phi - f \int_{-\infty}^{+\infty} \iint |\mathbf{u} - \mathbf{u}_1| \, \sigma(|\mathbf{u} - \mathbf{u}_1|) f(\mathbf{r}, \mathbf{u}_1, t) \, d\omega_1, \tag{12}$$

which is the Boltzmann equation written in our notation, and where Φ is determined by formula (8).

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named after A. A. Zhdanov

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Note: Figure translations are in progress. See original paper for figures.