ON ELASTIC DEFORMATIONS OF CONVEX SHELLS IN THE POSTCRITICAL REGION

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Abstract

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We consider such elastic deformations of a strictly convex shell under which its shape differs significantly from the initial one. We shall call them **postcritical**.

1. Postcritical elastic deformations of a geometrically non-bendable convex shell are inevitably accompanied by a loss of convexity and the appearance of regions of bulging. Convincing geometric considerations can be given in support of the fact that the true form of bulging of a convex shell under postcritical deformation, independently of the method of fixing at the boundary (clamping or hinged support), is close to the corresponding form of mirror bulging, which consists in the mirror reflection of a segment of the shell in the plane cutting it off.

2. The elastic deformation energy associated with bulging of the shell over a region G, bounded by a curve γ, we conventionally divide into two parts: U_G and U_γ. The energy U_G is associated with the "turning inside out" of the shell in the region of bulging. The energy U_γ is determined by significant local bending at the boundary of the region of bulging and by the stresses in the middle surface caused by this bending.

For small regions of bulging,

$$U_G = \frac{\pi E h \delta^3}{3(1-\mu^2)\sqrt{k_1 k_2}} \left\{(k_1 + k_2)^2 - 2(1-\mu)k_1 k_2\right\},$$

where h is the height of the mirror bulge; δ is the thickness of the shell; k_1 and k_2 are the principal curvatures of the shell at the center of the bulge; E is Young' s modulus; μ is Poisson' s ratio.

3. For a given region of bulging and a sufficiently small shell thickness, the energy U_γ, associated with significant local bending, is concentrated in a small neighborhood of the boundary of the bulge and depends essentially only on the geometric form of the shell along the line γ under mirror bulging, i.e. on the curvature $1/\rho$ of the curve γ and the angle α formed by the tangent planes of the shell along γ with the plane determining the

mirror bulging. For the magnitude of the energy U_γ, referred to a unit length of γ, for small α and correspondingly small δ, we find the expression

$$U = cE\delta^{5/2}\alpha^{5/2}\rho^{-1/2}, \qquad c = \frac{I_0}{12^{3/4}(1-\mu^2)^{3/4}} \quad (c = 0,12),$$

where I_0 is the minimum of the functional

$$I = \int_0^\infty ({v'}^2 + u^2)\, ds.$$

under the nonholonomic constraint $u' + v + v^2/2 = 0$ in the class of functions u, v satisfying the boundary conditions $u(0) = 0,\ v(0) = -1,\ u(\infty) = v(\infty) = 0.$

Hence, for small regions of bulging,

$$U_\gamma = \pi cE(2h)^{3/2}\delta^{5/2}(k_1 + k_2),$$

where $2h$ is the height of the mirror-image bulge.

4. We determine the dimensions of the bulging region G, for a given center of bulging P, by comparing the elementary work dA_G done by the external load and the change in the elastic deformation energy of the shell associated with bulging: $dU_G + dU_\gamma$. If the load q is continuous and, consequently, changes little in the bulging region, then in the case of a small region

$$dA_G = \frac{4\pi qh\, dh}{\sqrt{k_1 k_2}},$$

where $2h$ is the height of the bulge, and k_1 and k_2 are the principal curvatures at the center of bulging P.

From the equilibrium condition $dA_G = dU_\gamma + dU_G$, the following relation is obtained between the parameters $\xi = q/E\delta^2 K$ and $\eta = \sqrt{2h/\delta}$, which characterize the external load and the bulging:

$$\sqrt{K}\,\xi\eta^2 = 3cH\eta + \frac{1}{6\sqrt{K}}(4H^2 - 2(1-\mu)K),$$

where H is the mean curvature and K the Gaussian curvature at the center of bulging. The position of the center of bulging is determined from the condition of the maximum of the elastic deformation energy associated with bulging.

5. We assume that the stresses on the surface caused by the local bending at the boundary of the bulge are the most dangerous from the point of view of the strength of the shell. For the magnitude of these stresses, for a small bulging region, the formula obtained is

$$\sigma = \pm c' E (2h)^{1/2} \delta^{1/2} \sqrt{K}, \qquad c' = \frac{1}{2}\, 12^{1/4} (1-\mu^2)^{1/4} v'(0) \simeq 1,$$

where $v'(0)$ is the value, at $s = 0$, of the derivative of the function v realizing the minimum of the functional I (item 3).

6. It is easy to verify that all elastic states of the shell with bulging under the action of a continuous load q, independent of the deflections, are unstable, and, consequently, from any such equilibrium state the "snap-through" of the shell may begin. Snap-through of the shell stops because of the appearance of plastic deformations at the boundary of the bulging.

7. The so-called lower critical load q_k is defined as the lower bound of the loads q corresponding to stable equilibrium states of the shell with bulging. From the point of view of the scheme of the supercritical elastic state that we have constructed, the determination of the lower critical load is inevitably connected with consideration of inelastic deformations of the shell, since all elastic equilibrium states are unstable. We can assert only that the lower critical load is no greater than any load q corresponding to an elastic supercritical state. Thus, for it one obtains the estimate

$$q_k < \xi^* E \delta^2 K,$$

where

$$\xi^* = 3cc'(H\delta)\frac{E}{\sigma_e} + \frac{c'^2}{6}(4(H\delta)^2 - 2(1-\mu)K\delta^2)\left(\frac{E}{\sigma_e}\right)^2$$

(σ is the yield strength of the shell material).

8. With the aid of the results established in §§3, 4, and 5, a number of problems of practical interest can be solved. We shall give some of them:

1) What deformations involving buckling can be caused by a "concentrated" large force F, and what stresses associated with buckling arise in this case?

2) How does forced buckling of the shell affect the magnitude of the upper critical load?

3) How should rigid elements reinforcing the shell be arranged so that, for a given load, snapping-through of the shell is excluded?

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Note: Figure translations are in progress. See original paper for figures.