# ON THE QUESTION OF THE IRON CORE OF THE EARTH

!\[Fig. 1\](image)

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Fig. 1

Figure 1: Fig. 1

## Abstract

## Full Text

## GEOPHYSICS

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# ON THE QUESTION OF THE IRON CORE OF THE EARTH

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The Earth' s core is divided, according to seismological data, into two parts [1–4]: the outer core (zone $E$), extending from the boundary with the Earth' s mantle at depth $l = 2900$ km to depth $l \sim 5000$ km, and the inner core (zone $G$) from $l \sim 5000$ km to the center of the Earth $l \sim 6370$ km. The outer core is separated from the inner core by a transition region of order $\Delta l = 200$ km (zone $F$). The outer core is in the liquid state, since it does not transmit transverse waves with periods $\tau \sim 1$ sec.; consequently, it must have a viscosity $\eta \lesssim 10^{12}$ poise and be composed of a mechanically homogeneous substance. The temperature distribution in the core must be close to adiabatic, since at smaller temperature gradients convective currents leading to the origin of the Earth' s magnetic field would be impossible, whereas at gradients considerably larger than the adiabatic one, convective currents would lead to practically infinite thermal conductivity, and the temperatures would return to adiabatic values. The state of the inner core and of the transition zone $F$ is less clear [1–5]. Two independent and equally valid distributions of longitudinal-wave velocities for the Earth' s core are known (see Fig. 1). In zone $F$ the two distributions differ, and in Fig. 1 they are omitted [1–4].

**Fig. 1.** 1 –distribution of longitudinal-wave velocities according to Gutenberg; 2 –distribution of longitudinal-wave velocities according to Jeffreys; 3 –distribution of velocities in the liquid iron core according to formulas (3), (4); 4 –density distribution; 5 –pressure distribution; 6 –temperature distribution according to formula (10); crosses –adiabatic temperatures.

It is generally accepted that the substance composing the Earth' s core is in a metallic state. The question of the specific chemical composition of the core has not yet been resolved. According to a hypothesis put forward many decades ago, the core consists mainly of iron. Another debated poss–

A possibility is the hypothesis proposed by Lodochnikov and Ramsey, according to which the core consists of metallized silicates. Both these hypotheses currently permit experimental verification. In the present communication an attempt is made to carry out such a verification for the hypothesis of an iron core.

The equation of state of iron for the high-pressure phase was obtained in ([6]). For temperatures $T$ greater than the Debye temperature of iron $\theta$, it has the form

$$p = p_0(x) + \frac{3R}{M}\frac{\rho_0\gamma}{x}T + p_{\text{el}},$$

$$p_0(x) = x^{-2/3}\Sigma - K_2x^{-4/3}, \qquad \Sigma \equiv K_1e^{-bx^{1/3}} \equiv Ae^{b(1-x^{1/3})},$$

$$p_{\text{el}} = \frac{1}{2}\delta\rho_0x^{-1/3}\alpha g_{\text{el}}T^2, \qquad \delta = \left(\frac{\pi}{3}\right)^{2/3}\frac{k^2m_0N_0}{M\hbar^2}\left(\frac{M}{\rho_0}\right)^{2/3}, \qquad \alpha = Z^{1/s}\frac{m^*}{m_0}, \tag{1}$$

$$\alpha = \alpha_0e^{B(x^{1/3}-1)}, \qquad g_{\text{el}} = \frac{2}{3} + \frac{d\ln\alpha}{d\ln x} = \frac{2}{3} + \frac{B}{3}x^{1/3},$$

where $p$ is pressure; $x = V/V_0$ is the relative volume; $V_0 = 1/\rho_0$ is the volume under normal conditions; $k$ is Boltzmann' s constant; $R$ is the gas constant; $\hbar$ is Planck' s constant divided by $2\pi$; $m_0$ is the mass of a free electron; $m^*$ is the effective mass of a conduction electron; $N_0$ is Avogadro' s number; $Z$ is the number of conduction electrons per atom; $A, K_2, b, \alpha_0, B$ are constants determined in ([6]) with the aid of experimental data:

$$A = 9.4389{\cdot}10^5 \text{ bar}, \qquad K_2 = 1.0740{\cdot}10^6 \text{ bar}, \qquad b = 7.7845, \qquad \alpha_0 = 1.72{\cdot}10,$$

$$B = 9.86 \quad (1 \text{ bar} = 10^6 \text{ dyn/cm}^2).$$

The Grüneisen parameter $\gamma$ depends only on volume; it was determined by the Dugdale-MacDonald formula on the zero isotherm ([6])

$$\gamma = \frac{1}{3} - \frac{x}{2}\frac{\partial^2(px^{2/3})/\partial x^2}{\partial(px^{2/3})/\partial x} = \frac{1}{6}\frac{(bx^{1/3})^2\Sigma - 6K_2x^{-2/3}}{(bx^{1/3})\Sigma - 2K_2x^{-1/3}}. \tag{2}$$

With the aid of (1) it is easy to obtain the quantity $\Phi = K_s/\rho$, where $K_s$ is the adiabatic bulk modulus of incompressibility and $\rho$ is density:

$$\Phi = \frac{K_s}{\rho} = \frac{1}{3\rho_0}(2x^{1/3} + bx^{2/3})\Sigma - \frac{4}{3\rho_0}K_2 x^{-1/3} + \frac{3R}{M}\gamma\left(1 - \frac{d\ln\gamma}{d\ln x}\right)T +$$

$$+\frac{x}{\rho_0}\left(1 - g_{\text{el}} - \frac{d\ln g_{\text{el}}}{d\ln x}\right)p_{\text{el}} + \frac{(\gamma 3R/M + g_{\text{el}}\delta\alpha x^{2/3}T)^2}{3R/M + \delta\alpha x^{2/3}T}T. \tag{3}$$

The velocity of longitudinal waves in the liquid core $v_p$ is equal to

$$v_p = \Phi^{1/2}. \tag{4}$$

The adiabatic temperature gradient is determined in the standard manner ([7]) and, after elementary transformations, takes the form

$$\frac{dT}{dl} = \frac{\bar{g}\chi T}{c_p} = \frac{\bar{g}T}{\Phi}\frac{\gamma 3R/M + g_{\text{el}}\delta x^{2/3}\alpha T}{3R/M + \delta\alpha x^{2/3}T}, \tag{5}$$

where $\bar{g}$ is the acceleration of gravity; $c_p$ is the heat capacity at constant pressure; $\chi$ is the coefficient of thermal expansion

$$\chi = \frac{1}{x}\left(\frac{\partial x}{\partial T}\right)_p = \frac{1}{K_T}\left(\frac{\partial p}{\partial T}\right)_x = \frac{(3R/M)(\rho_0\gamma/x) + \delta\rho_0 x^{-1/3}\alpha g_{\text{el}}T}{K_T};$$

$$K_T = \frac{1}{3}(2x^{-2/3} + bx^{-1/3})\Sigma - \frac{4}{3}K_2 x^{-4/3} + \tag{6}$$

$$+\frac{3R}{M}\frac{\rho_0\gamma}{x}\left(1 - \frac{d\ln\gamma}{d\ln x}\right)T + \left(1 - g_{\text{el}} - \frac{d\ln g_{\text{el}}}{d\ln x}\right)p_{\text{el}};$$

$K_T$ is the isothermal modulus of volume incompressibility; the heat capacity at constant volume is

$$c_v = \frac{3R}{M} + \delta a x^{2/3}T. \tag{7}$$

Let us now consider the data on the Earth' s core obtained by geophysical methods. We shall regard the entire core as homogeneous, and the increase in velocities on going from the outer core to the inner core as being associated with the transition of seismic waves from liquid regions, with $v_p^2 = K_s/\rho$ (zone $E$), to solid regions, with

$$v_p^2 = \frac{K_s + \frac{4}{3}\mu}{\rho}$$

(zone $G$), where $\mu$ is the unrelaxed value of the shear modulus (for more detail see (5)). From the theory of gravitational attraction it follows ([1,3,4])

$$\frac{dp}{dl} = \bar{g}\rho, \qquad \bar{g} = \frac{Gm}{r^2}, \qquad m = 4\pi \int_0^r \rho(y) y^2 \, dy, \tag{8}$$

and, for an adiabatic change of temperature, the equation for the change of density with depth has the form ([3])

$$\frac{d\rho}{dl} = \frac{Gm\rho}{r^2 \Phi}, \tag{9}$$

where $G$ is the gravitational constant and $r$ is the distance from the center of the Earth. Using the quantity $\Phi = v_p^2$, determined by seismologists, and choosing an initial value of the density at the surface of the core, one can, by numerical integration of equations (8) and (9), determine the distribution of density $\rho$ (and hence also $p$ and $\bar{g}$) for the whole core. Consideration of various models of the distribution of density and pressure, carried out by Bullen ([3]), showed that the pressure at the mantle-core boundary is $p_{2900} \sim (1.35\text{–}1.4) \cdot 10^6$ atm, while the value $\Phi_{2900}^{1/2} = v_p \sim 8$ km/sec. Using these initial conditions, we determined, by means of (1) and (3), $x_{2900} = \rho_0 / \rho_{2900}$ and $T_{2900}$. It turned out that $\rho_{2900} \simeq 9.7$ g/cm$^3$, $T_{2900} \simeq 6750$°K. If, as the initial conditions, one takes $p_{2900} \sim 1.4 \cdot 10^6$ atm, $\Phi_{2900}^{1/2} = v_{p\,2900} \sim 7.85$ km/sec (which lies within the accuracy limits for $v_p$ as determined by seismological methods), then one obtains $\rho_{2900} \simeq 10.2$ g/cm$^3$, $T_{2900} \simeq 6080$°K. The latter values of $\rho_{2900}$ and $T_{2900}$ were adopted as the initial ones.*

Next, with the aid of (8) and (9) and with $\rho_{2900} = 10.2$, the distribution in a homogeneous core of the density $\rho_{\mathrm{c}}$ and pressure $p_{\mathrm{c}}$ was determined (see Fig. 1) (and also $\bar{g}$). We note that the density distribution obtained is very close to the upper boundary of the density distribution in the core according to M. S. Molodenskii ([8]). Knowing $\rho_{\mathrm{c}}$ (and thus also $x$) and $p_{\mathrm{c}}$, one can determine the temperatures in the core by solving equation (1) for $T$:

$$T = \frac{\left\{ \left( \frac{3R}{M} \frac{\gamma\rho_0}{x} \right)^2 + 2\delta\rho_0 x^{-1/3} a g_{\mathrm{el}} \left[ p_{\mathrm{c}} - p_0(x) \right] \right\}^{1/2} - \frac{3R}{M} \frac{\gamma\rho_0}{x}}{\delta\rho_0 x^{-1/3} a g_{\mathrm{el}}}. \tag{10}$$

The temperatures calculated from (10) (see Fig. 1) were substituted, together with the corresponding values of $x$, into (3) and (4). The velocity distribution for a homogeneous liquid iron core according to (3) and (4) is presented in Figs. 1 and 3. Finally, equation (5) was numerically integrated at an initial temperature $T_{2900} = 6080$°K; the adiabatic temperatures obtained in this way are indicated in Fig. 1 by crosses. These temperatures are close to the temperatures determined from formulas (10). The calculated distribution of $v_p$ also proved close to that determined experimentally—

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* Use of the initial values $\rho_{2900} \sim 9.7$ g/cm$^3$ and $T_{2900} \sim 6750$°K leads to the same qualitative picture, but the temperature distribution is shifted upward by 500–700°K.

-tally (see Fig. 1) (only the results obtained for the outer, "liquid" core should be compared).

In summary, we come to the conclusion that the available experimental data do not qualitatively contradict the hypothesis of the Earth' s iron core. However, the following difficulties are already apparent: 1) the values of $v_p$, determined by seismological methods, increase toward the center of the Earth more steeply than the values of $v_p$ for iron obtained on the basis of laboratory observations with the aid of formulas (3) and (4); the greatest discrepancy between these quantities reaches $\sim 5\%$ near the inner core; 2) the increase in temperature from the boundary of the core with the mantle to the center by approximately $6 \cdot 10^3$ °K is rather large; the existence of temperatures $\sim 1.2 \cdot 10^4$ °K at the center of the Earth, proceeding from theories of the cold formation of the Earth, seems incomprehensible.

Both of the difficulties noted are connected with the fact that the increase of pressure with decreasing volume on the zero isotherm of iron occurs more slowly than the analogous increase of pressure in the core. This leads to the fact that in formula (10) the difference $\rho_z - \rho_0(x)$ increases noticeably from the periphery toward the center of the core. Compensation of this difference by thermal pressure leads to an increase in temperature. Numerical analysis has shown that increasing the constant $b$ in Davydov' s equation ([1]) for the zero isotherm of iron to values $b \sim 8.3$ would eliminate both difficulties noted above. This may occur if it turns out that the shock adiabat of iron ([9]) in the pressure region $\sim 1 \cdot 10^6$ atm, which is important for obtaining the zero isotherm, goes $7 \div 10\%$ more steeply than was obtained by Altshuler and co-workers, or else upon adding to iron a small amount of nickel, provided that the zero isotherm of such an alloy has a steeper course.

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*Note: Figure translations are in progress. See original paper for figures.*