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Abstract

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ON THE SYSTEMATIC CHARACTER OF DEVIATIONS FROM THE LAWS OF PLASTICITY

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The propositions adopted for conditions of simple loading—that the relation between the intensities of stresses and strains is constant, and that the corresponding deviators are similar (when they are coaxial)—are confirmed in experimental verification only approximately*. The absence of complete similarity of the stress and strain deviators has been noted by many investigators (([1], [2]) and others), and the observed violations of similarity always have one and the same character: at values of the parameter of the form of the stress deviator $\mu = \pm 1$ and $\mu = 0$ there are no violations, i.e., the parameter of the form of the strain deviator ν proves to be equal to the parameter μ, while in the remaining cases violations of similarity occur for which $|\nu| < |\mu|$ (see the curve in Fig. 1). Thus, the existence of nonrandom deviations from similarity of deviators, constant and manifested in the same way, raises no doubts, and there is no disagreement on this point in the literature. The situation is otherwise with regard to how close to reality is the condition of constancy of the dependence between the stress intensity σ_i and the strain intensity ε_i. On this question there is no single point of view; moreover, the majority of investigators (([2]–[6]) and others) hold the opinion that, for pure metals and for a wide range of alloys used in engineering, the relation between σ_i and ε_i does not depend on the type of stress state, i.e., plastic deformation in the coordinates σ_i–ε_i for each metal is described by a single curve. In this case, the deviations from the single curve obtained from the results of one experiment or another are usually attributed to possible inhomogeneity of the specimens, anisotropy of the material, and imperfections in the arrangement and execution of the experiments. However, one cannot agree with this view. A study of the literature data compels one to assume the existence of noticeable regular deviations also from the condition of constancy of the relation between the intensities of stresses and strains. Not to mention certain changes in the resistance to plastic deformation depending on the mean normal stress, noted in a number of works ([6], [7]), the form of the stress deviator

Fig. 1

Figure 1: Fig. 1

also undoubtedly exerts an influence. Since the question of the influence of the latter has scarcely been covered in the literature** and is not clear, the authors carried out a special study of it by means of experiments arranged as rigorously as possible.

Technically pure nickel was used as the test material. The specimens were sections of cold-drawn tubes with an outside diameter of 8.1 mm, a wall thickness of 0.2 mm, and a gauge length of 165 mm. Within a single specimen, fluctuations in the diameter did not exceed ±0.02 mm, and the wall thickness varied by no more than ±0.005 mm. The specimens were annealed at a temperature of 860° with cool—

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* In the geometrical representation in the combined spaces of the stress and strain deviators, the indicated propositions are described in the form of two coinciding radius vectors in direction, whose moduli are connected by a one-to-one dependence.

** The question of the influence of the form of the stress deviator was considered only in work ([8]).

in a furnace. The tests were carried out on an apparatus similar to that described in ([9]), which made it possible, with great accuracy, to ensure a homogeneous stressed state in the wall of a tubular specimen under the action, in various combinations, of a tensile force, a twisting moment, and internal pressure.

Longitudinal strains and angles of twist were measured by means of Martens mirror instruments, and for measuring transverse strains a special instrument was designed ([10]). At each loading stage of the specimen, readings from the instruments were taken after holding for 7–8 min, necessary for the creep to subside. Plastic strains were found from the differences between the total and elastic strains, determined from unloadings. Radial plastic strains were calculated from the condition of constancy of volume, which, when checked by the method of hydrostatic weighing, was confirmed (in the range of strains realized) with very high accuracy. The processing of the results was carried out in true stresses and strains.

Fig. 1

Special experiments were carried out to check the degree of isotropy of the material. These experiments consisted of comparative tests of specimens under one and the same type of stressed state (characterized by $\mu = 0$ and $\sigma_{\text{cp}}/\sigma_i = 0.57$), but with different directions of the principal axes of the stress tensor. These tests showed that the material possesses a certain anisotropy, which leads

Fig. 2

Figure 2: Fig. 2

to the greatest discrepancy between the strain curves corresponding to the axial and tangential directions of the greatest principal stress. The values of σ_i from the corresponding curves (in the range of values of ε_i from 5 to 10%) deviate from the mean by $\pm 2.5\%$, and some fraction of these deviations must be attributed to the influence of specimen inhomogeneity.

Fig. 2

The main experiments consisted of tests of specimens under conditions of proportional loading for various types of stressed state, covering the entire range of variation of the parameter μ of the type of stress deviator from -1 to $+1$, while the ratio of the mean normal stress to the stress intensity varied within the limits from 0.33 to 0.66. For each type of stressed state, several specimens were tested, which were tested with different directions of the greatest principal stress; this made it possible, by averaging the test results, practically to eliminate from the strain curves the influence both of anisotropy of the material and of inhomogeneity of the specimens. Such averaged curves are presented in Fig. 2. Rather considerable discrepancies were obtained between the curves. Since the mean normal stress, varying within the indicated small limits, could exert only a very small

effect, the discrepancies obtained must be attributed only to the difference in the form of the stress deviator. Under these conditions the curves should be situated in a definite dependence on the absolute value of μ*. This is indeed observed: the curves are arranged successively higher the larger $|\mu|$ is; moreover, in the range of changes of ε_i from 5 to 10%, the difference between the largest value of σ_i at $\mu = \pm 1$ and its smallest value at $\mu = 0$ is about 12% relative to the mean values of σ_i.

It is easy to see that the discrepancies obtained between the curves represent deviations from the condition of constancy of σ_i, corresponding to the fourth theory of strength, in the direction of approaching the condition of the third theory of strength, since equal intensities of plastic strains prove to be attainable under stress states enclosed between the boundaries $\sigma_i = C$ and $\sigma_1 - \sigma_3 = C$ (where C is equal to the coinciding values of σ_i and $\sigma_1 - \sigma_3$ at $\mu = \pm 1$). This situation can be illustrated visually on the deviatoric plane in the three-dimensional space of principal stresses. In Fig. 3, on such a plane, the construction has been carried out for the strain-intensity value $\varepsilon_i = 8\%$. The circle and the regular hexagon shown in the figure correspond respectively to the criteria of the fourth and third theories, while the solid curve drawn between them corresponds to the experimental data obtained. For other values of strain intensity the character of the curve remains the same.

Fig. 3

Fig. 3

Figure 3: Fig. 3

Similar results were also obtained by other investigators, who, as noted, did not give them due attention. Thus, in the work of Davis ([2]), who carried out experiments on copper, it is clearly seen from the positions of the points plotted in octahedral coordinates that the test diagrams for $\mu = +1$ and $\mu = -1$ are close to one another, but at the same time pass considerably above the diagrams corresponding to $\mu = 0$. The discrepancy in the ordinates of these diagrams is approximately 9%. The points corresponding to tests at other values of μ lie in the interval between the diagrams for $\mu = \pm 1$ and $\mu = 0$. If, from these data, a curve is marked on the deviatoric plane, it will approximately have the form shown in Fig. 3 by the dashed line. Analogous conclusions follow from the data presented in the work of Osgood and Washington ([4]), who carried out tests on alloy 24 S-T**.

In the experiments described, the plastic strains developed with deviations from the similarity of the stress and strain deviators. These deviations had the ordinary character described above (see Fig. 1). It is not difficult to see that the indicated constantly observed deviations also correspond to an approach to the concepts of the third theory of strength. Indeed, according to this theory, plastic deformation should occur

* In reality, according to the results of tests at values of μ differing only in sign, curves were obtained that were very close, but not fully coincident; their ordinates were then averaged.

** In work ([8]), in tests under biaxial and triaxial compression, a different arrangement of the curves was obtained, with a minimum value of σ_i at $\mu = -0.5$. This result may be connected with instability of the structure of the bronzes tested, as well as with other working conditions of the materials during the compression tests that were carried out.

only through shears along the planes of action of the greatest tangential stresses, and therefore the deformation ε_2 must be absent. By the condition of constancy of volume this leads, independently of the value of μ, to the equality $\nu = 0$. The only exceptions are stress states characterized by the values $\mu = 1$ and $\mu = -1$, for which, in view of the equality of two principal stresses, one obtains $\varepsilon_2 = \varepsilon_1$ for $\mu = 1$ and $\varepsilon_2 = \varepsilon_3$ for $\mu = -1$. Correspondingly, for ν the same values are obtained as for μ, i.e. ± 1. In view of what has been set forth, the relation between ν and μ according to the third theory of strength will be represented on the graph (Fig. 3) by the broken line *abcd*. Thus, the experimental curve obtained indeed represents deviations from similarity of the deviators (the straight line *ad*) in the direction of approximation to the conditions of deformation in accordance with the third theory of strength.

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Note: Figure translations are in progress. See original paper for figures.