GEOPHYSICS

A. S. GURVICH

1960

SovietRxiv

Abstract

Full Text

GEOPHYSICS

A. S. GURVICH

MEASUREMENT OF THE SKEWNESS COEFFICIENT OF THE DISTRIBUTION OF THE VELOCITY DIFFERENCE IN THE SURFACE LAYER OF THE ATMOSPHERE

(Presented by Academician A. N. Kolmogorov, 7 VI 1960)

In the theory of locally isotropic turbulence developed by A. N. Kolmogorov ([1,2]), an important role is played by the second-order and third-order structure functions D_{ik} and D_{ikl}:

$$D_{ik} = \overline{(u_{i1} - u_{i2})(u_{k1} - u_{k2})}, \qquad D_{ikl} = \overline{(u_{i1} - u_{i2})(u_{k1} - u_{k2})(u_{l1} - u_{l2})};$$

u_{i1} is the i-component of the velocity at the point M_1; u_{i2} is the i-component of the velocity at the point M_2; the bar above denotes averaging. To investigate the properties of structure functions it is convenient to use a special coordinate system in which the axis x_1 coincides with the vector $\vec{r} = \overrightarrow{M_1 M_2}$, and to consider two components of the velocity difference: $\Delta u_l = u_{l1} - u_{l2}$, the projection of $\overrightarrow{\Delta u}$ on the direction $\vec{r}$, and $\Delta u_n = u_{n1} - u_{n2}$, the projection of $\overrightarrow{\Delta u}$ on the perpendicular direction. For distances $l_0 \ll r \ll L$, the Kolmogorov-Obukhov “two-thirds law” is valid for the second-order structure functions:

$$D_{ll} = {}^3\!/_4\, D_{nn} = {}^3\!/_4\, c^2 (\varepsilon r)^{2/3}, \qquad D_{nl} = 0,$$

where ε is the rate of energy dissipation; c is the structure constant; $l_0 = \sqrt[4]{\nu^3/\varepsilon}$ is the inner scale of turbulence; ν is the kinematic viscosity; L is the outer scale of turbulence.

The second- and third-order structure functions are related by Kolmogorov’ s dynamical equation

$$D_{lll}(r) - 6\nu \frac{\partial D_{ll}(r)}{\partial r} = -\frac{4}{5}\,\varepsilon r.$$

For $r \gg l_0$, the viscosity term may be omitted, and then a relation between the skewness coefficient $S_l = D_{lll}/(D_{ll})^{3/2}$ and the structure constant c^2 can readily be found:

$$c^2 = \frac{4}{3}\left(-\frac{4}{5S_l}\right)^{2/3}. \qquad (1)$$

Measurements of second-order structure functions have been carried out both in wind tunnels and in the atmosphere by a number of investigators, and the validity of the "two-thirds law" has been verified on a large body of experimental material. Work on measuring third-order structure functions is quite sparse. Measurement of the skewness of the distribution of the velocity difference at two points was performed by Townsend ([3]) in a wind tunnel. The Reynolds number in his measurements was $\mathrm{Re} = 4.4 \cdot 10^4$; at larger Re it was not possible to carry out measurements.

The values of the skewness coefficient S_l obtained by Townsend for $0 \leq r/M \leq 0.2$ (M is the mesh size of the grid in the wind tunnel) are constant within the limits of error, and on average $S_l = -0.38$. When r/M is increased to 0.8, the skewness decreases somewhat, $S_l(0.8) = -0.25$. Stewart' s measurements (4), also in a wind tunnel, showed a different dependence of S_l on r: at the largest Reynolds-number values in his measurements, $\mathrm{Re} = 4.2 \cdot 10^4$, when r/M was varied from 0.25 to 0.7, S_l changed from -0.17 to -0.11 and differed strongly from the value of S_l at zero, $S_l(0) = -0.34$. Stewart, however, notes, on the basis of estimates of the experimentally obtained data, that even at the largest Re numbers in his experiments the condition under which A. N. Kolmogorov' s hypothesis is valid was not fulfilled—namely, that there exists an interval of scales in which the energy of turbulence is transferred from larger scales to smaller ones, while energy dissipation occurs only at the very smallest scales*.

To our knowledge, direct measurements of third-order structure functions in the atmosphere have not been carried out. A. M. Obukhov gave an estimate of the value of S_l (5), processing the results of measurements of second-order structure functions in the surface layer of the atmosphere. The value he obtained was $S_l = -0.8$.

We measured the skewness coefficient of the distribution of the velocity difference at two points in an open, level steppe in September 1959. To obtain instantaneous values of the wind velocity, two acoustic microanemometers (6) were used; their sensors were located at a height $z = 1.8$ m and could be oriented so that either Δu_l or Δu_n was measured in the horizontal plane. Since acoustic microanemometers are linear velocity sensors, the voltages from their outputs could be fed to a subtracting circuit, which produced a signal proportional to the velocity difference. To compute the skewness coefficients it is necessary to have simultaneous values of the mean cubes and variances of the velocity difference. To measure the variance $\overline{\Delta u_l^2}$ or $\overline{\Delta u_n^2}$, an electrodynamic multiplier with feedback (e.d.m.), similar to that described in (7), was used, with which the mean square of the voltage at the output of the subtracting circuit was measured. The averaging time was determined by the time constant $T = 100$ s of the integrating cell at the output of the e.d.m. To obtain the mean cubes $\overline{\Delta u_l^3}$

and $\overline{\Delta u_n^3}$, a second e.d.m. and a correlometer, analogous to that described in (8) but without an averaging device at the output, were used. The signal from the correlometer, proportional to the instantaneous square of the velocity difference, was fed to one of the inputs of the e.d.m., while to its second input a signal was fed directly from the subtracting circuit. Thus, at the output of the second e.d.m. we had the mean (over 100 s) cube of the velocity difference with some constant coefficient. The proportionality coefficients were determined by calibrating the e.d.m. and the correlometer with voltage from an audio-frequency generator. The multipliers made it possible to measure products of signals with frequencies from 0.002 to 800 Hz.

We determined the values

$$S_l = \frac{\overline{\Delta u_l^3}}{\left(\overline{\Delta u_l^2}\right)^{3/2}} \quad \text{and} \quad S_n = \frac{\overline{\Delta u_n^3}}{\left(\overline{\Delta u_n^2}\right)^{3/2}}.$$

The latter quantity, in a locally isotropic flow, must vanish by considerations of symmetry. Its deviation from zero may serve as an additional check on the correct operation of the entire apparatus as a whole. This is especially necessary when measuring mean cubes, since a small uncontrolled nonlinearity of the measuring instruments and sensors can lead to a significant distortion of the measured values of the skewness coefficient. Averaging over 100 s when measuring the coefficient–

* In (4) it is not stated how the influence of the sensor dimensions on the measurement results was taken into account, although the dimensions were comparable with the distances r.

of the asymmetry coefficients proved insufficient to obtain statistically stable results. Additional averaging was obtained by recording the readings of the instruments regularly every minute and subsequently taking the arithmetic mean of these data. The measurement time τ ranged from 5 to 24 min. For the mean values $\overline{\Delta u_l^3}$ and $\overline{\Delta u_l^2}$, the root-mean-square deviation of the arithmetic mean was calculated.

Table 1

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
r, cm	30	30	50	50	25	25	25	50	50	50	50	50	50	50
τ, min	5	10	20	12	9	20	10	24	20	20	11	9	20	9
$\mathrm{Ri} \cdot 10$	−2.4	−3.4	–	–	−3.8	−2.8	−3.0	−2.5	−3.6	−6.6	−3.2	−10.8	−8.2	−6.5
$\mathrm{Re} \cdot 10^{-5}$	6.4	5.8	–	–	2.2	2.4	2.0	2.5	1.9	1.9	2.2	1.6	1.9	1.8

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$S_n \cdot 10$	+0.7	+0.6	+0.4	−0.3	–	–	–	–	–	–	–	–	–	–
$S_l \cdot 10$	–	–	–	–	−4.0	−5.5	−4.1	−6.2	−5.4	−3.8	−1.9	−2.2	−3.3	−5.0
$S_l \cdot 10$	–	–	–	–	± 0.6	± 0.7	± 0.9	± 0.9	± 1.5	± 0.8	± 0.6	± 0.9	± 0.6	± 0.12

Table 1 gives the values of S_n and S_l obtained by us, as well as certain dimensionless characteristics of the flow: the Reynolds number $\mathrm{Re} = \overline{u}z/\nu$ and the Richardson number

$$\mathrm{Ri} = \frac{g}{T_0}\frac{\partial \overline{T}(z)/\partial z}{(\partial \overline{u}(z)/\partial z)^2},$$

where $\overline{u}(z)$ is the mean velocity, $\overline{T}(z)$ is the mean temperature at height z, T_0 is the mean temperature of the surface layer, and $g = 981$ cm/sec^2. Also given are the distance r between the points at which the sensors were located and the averaging interval τ.

From the data of Table 1 the mean values of S_l and S_n and their root-mean-square errors were calculated. For S_n the mean value proved to be $S_{n\,\mathrm{mean}} = 0.03 \pm 0.02$. For $r = 25$ cm ($r/z = 0.14$), the mean value is $S_{l\,\mathrm{mean}} = -0.45 \pm 0.05$, and for $r = 50$ cm ($r/z = 0.28$), $S_{l\,\mathrm{mean}} = -0.40 \pm 0.06$. The smallness of S_n indicates the normal operation of the apparatus, in particular sufficient linearity. The values of $S_{l\,\mathrm{mean}}$ for $r = 25$ cm and $r = 50$ cm agree within the error of measurement, which confirms the assumption that, in a locally isotropic turbulent flow, S_l depends only weakly on r. The values of S_l measured by us are very close to those obtained by Townsend under quite different experimental conditions.

Institute of Atmospheric Physics
Academy of Sciences of the USSR

Received
10 February 1960

References

1. A. N. Kolmogorov, *Dokl. Akad. Nauk SSSR*, **30**, No. 4 (1941).

2. A. N. Kolmogorov, *Dokl. Akad. Nauk SSSR*, **32**, No. 1 (1941).

3. A. A. Townsend, *Proc. Cambr. Phil. Soc.*, **44**, No. 4, 560 (1948).

4. R. W. Stewart, *Proc. Cambr. Phil. Soc.*, **47**, No. 2, 146 (1951).

5. A. M. Obukhov, *Izv. Akad. Nauk SSSR, Geophys. Ser.*, No. 3 (1951).

6. A. S. Gurvich, *Akust. Zhurn.*, No. 3 (1959).

7. G. Korn and T. Korn, *Electronic Analog Computers*, Moscow, 1955, p. 241.

8. V. M. Bovsheverov, A. S. Gurvich, V. I. Tatarskii, and L. R. Tsvang, *Proceedings of the Conference on Research on the Twinkling of Stars*, Moscow–Leningrad, 1959, p. 26.

Note: Figure translations are in progress. See original paper for figures.