Physics

V. G. Kadyshevsky

1960

SovietRxiv

Abstract

Full Text

Physics

V. G. Kadyshevsky

On the Question of the Mass Spectrum and the Fundamental Length in Field Theory

(Presented by Academician N. N. Bogolyubov, December 28, 1959)

§ 1. Modern field theory contains no constant with the dimension of length, and therefore the masses of elementary particles cannot be calculated. However, some information about the mass spectrum can already be obtained. Namely, using the properties of the automorphism group, one can show that if the system of all one-particle state amplitudes transforms according to some irreducible representation of the Lorentz group, then the mass in this system is a continuous parameter ([1]).

We shall denote the inhomogeneous Lorentz group by L, and its element by (Λ, a), where Λ is a 4-rotation and a is a 4-translation. The group property $(N, b)(M, c) = (\Lambda, a)$, in matrix form, evidently has the form

$$a = b + Mc, \qquad \Lambda = MN. \tag{1}$$

The transformation

$$a_0 = \nu a, \qquad \Lambda^0 = \Lambda, \tag{2}$$

where ν is any real number, taking (1) into account, is an automorphism of L. Let $\{|p_{(n)}\rangle\}$, where $p_{(n)}^2 = m_n^2$, be the system of all possible one-particle states. It is well known that the unitary representation $T(a)$ of the translation subgroup of L, according to which these vectors transform, is

$$T(a)|p_{(n)}\rangle = e^{-i(p_{(n)},\, a)}|p_{(n)}\rangle. \tag{3}$$

Let us perform the transformation (2) on L and find the form of the representation $T^0(a)$:

$$T(a^0) = T(\nu a) = T^0(a),$$

or

$$e^{-i(p_{(n)},\, a^0)} = e^{-i(p_{(n)},\, \nu a)} = e^{-i(\nu p_{(n)},\, a)} = e^{-i(p',\, a)}.$$

Since the representations $T(a)$ and $T^0(a)$ realize one and the same basis in the space of one-particle amplitudes, there must necessarily exist a state $|p'\rangle$ for which ${p'}^2 = \nu^2 m^2$. Thus, by virtue of the arbitrariness of ν, the continuity of the particle masses of the system $\{|p_{(n)}\rangle\}$ is proved.

§ 2. Since the continuity in ν is an invariant property of the group L, in order to construct a discrete mass spectrum in the theory it is necessary to replace L by some new group. We shall do this in the simplest and most economical way. Since $\Lambda^0 = \Lambda$, we shall not touch the pure 4-rotations, but shall concern ourselves only with translations

$$x^{i'} = x^i + a^i. \tag{4}$$

Evidently, there are only two possibilities for modifying (1^0) or generalizing (2^0) the transformation (4).

1°. The points x^i form a countable set, a^0 and a are multiples of some "quantum of space" l. In this case $v = a^0/a$ will be discrete, but this variant must nevertheless be abandoned, since any theory with quantized space-time as a whole is unsatisfactory.

2°. The transformations (4) of translations of the 4-world are a subgroup of the translation group of a space of a higher number of dimensions, for example, the 5-world. The additional 5th dimension must possess such a property as will allow one to use only the discrete parameter v and thus have a discrete mass spectrum. We must then determine in what way the 5th dimension can be introduced most reasonably, and how to endow it with the indicated property.

§ 3. If particles of different masses are interpreted as different mass states of a single material field, then the mass m itself may be regarded as a dynamical variable of this field. It is then natural to introduce the 5th dimension as the coordinate canonically conjugate to m, and to write the equations of motion of the fields in the form

$$\left(i\frac{\partial}{\partial x} + i\frac{\partial}{\partial x_5}\right)\psi = 0, \qquad \left(\Box + \frac{\partial^2}{\partial x_5^2}\right)\varphi = 0. \tag{5}$$

Since in (5) there is no interaction, the mass spectrum will be discrete only under the condition of periodicity of the fields ψ and φ in x_5, i.e. the 5th coordinate must be closed with some period l (2). In this case we have $m_n = 2\pi n/l$, where $n = 0, 1, 2, \ldots$. If one chooses $l/2\pi = 2r_0 = 5.6 \cdot 10^{-13}$ cm (r_0 is the "classical radius" of the electron), then we obtain the final formula for the masses ([2,3])

$$m_n = \frac{137}{2} n m_e \tag{6}$$

(m_e—the mass of the electron).

Putting in (6) $n = 0; 3; 4; 14; 27, ...,$ we find mass values (in units of m_e): $0; 205.5; 274; 959; 1849.5$, which are close to the masses of the photon, μ-, π-, K-mesons, the nucleon, etc. In this scheme the electron must have a purely field mass. It is also curious that in formula (6), for the particles written out, even n correspond to bosons, odd n to fermions. If this correspondence is considered obligatory for all intermediate and heavier particles, then it becomes convenient to use a Fourier expansion in the 5th coordinate.

Remark. We obtained the discrete mass spectrum by considering the 5th coordinate closed. It can be proved that the parameter v of the automorphism (2) is then also discrete.

§ 4. The 5th dimension introduced above may be called an analogue of the particle' s proper time (s), since in the usual theory the quantities m and s possess a large share of the symmetry characteristic of canonically conjugate variables. Indeed, if $A = -m \int_{s_0}^{s} ds$ is the classical action integral, then, along with the relations $E = -\partial A/\partial t$, $\mathbf{p} = \partial A/\partial \mathbf{x}$, one may write the equality $m = -\partial A/\partial s$. Moreover, for all particles always $m^2 \geq 0$ and $s^2 \geq 0$, and, if $m^2 = 0$, then $s^2 = 0$. In the limit of "geometrical optics" ($\hbar \to 0$), particles in the 5-world propagate along the characteristic cone $t^2 - \mathbf{x}^2 - x_5^2 = 0$, i.e. in this case x_5 is simply equal to s. Since m and S are internal parameters of the particle, x_5 too should naturally be regarded not as some additional dimension of space, but as an internal degree of freedom of matter located at the point $(\mathbf{x}, t)$. Moreover, the 5th coordinate, in contrast to the variables x, y, z, has a microscopic domain of variation.

§ 5. Since always $x_5 \leq l$, for $|\mathbf{x}|$, $t \gg l$ in the square of the 5-distance $S^2 = t^2 - \mathbf{x}^2 - x_5^2$ one may neglect the last term; the 5-theory thereby passes into the 4-theory, and the period l plays the role of the "fundamental length." At small intervals (high energies) the requirements of the 5-theory reduce

to the simultaneous consideration of all possible mass states of the field ([4]), and hence of the entire manifold of interactions. Taking into account the smallness of the 5th coordinate in comparison with most values of x, t, we may average the 5-interval over x_5; as a result one obtains the 4-form $s^2 = x^2 - a^2$ (here $a \sim l$), which must be regarded as more suitable for describing space-time than the ordinary interval $s^2 = x^2$. As shown in ([5]), in a theory with the interval $s^2 = x^2 - a^2$ there are no divergences, but there do exist indefinite densities of certain physical quantities and an indefinite metric in the Hilbert space of states. Within the framework of the exposition given above, the latter circumstance is due to the illegitimacy of averaging over x_5 for small x^2.

The author expresses his deep gratitude to the participants of the seminar directed by N. N. Bogolyubov, and to M. A. Markov, for fruitful discussion.

Moscow State University
named after M. V. Lomonosov

Received
25 XII 1959

CITED LITERATURE

1. E. Wigner, *Ann. of Math.*, **40**, 149 (1939).
2. P. Kard, *ZhETF*, **27**, 263 (1954).
3. Nambu, *Progr. Theor. Phys.*, **7**, 595 (1952).
4. Yu. B. Rumer, *Investigations in Pentoptics*, Moscow, 1956, p. 65.
5. M. A. Markov, *Nucl. Phys.*, **10**, 140 (1959); A. A. Komar, M. A. Markov, *Nucl. Phys.*, **12**, 190 (1959).

Note: Figure translations are in progress. See original paper for figures.