ON EXTREMAL CONTROL OF INERTIAL AND UNSTABLE OBJECTS

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Fig. 1

Figure 1: Fig. 1

Abstract

Full Text

CYBERNETICS AND CONTROL THEORY

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ON EXTREMAL CONTROL OF INERTIAL AND UNSTABLE OBJECTS

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1. Let us consider an extremal-control system (e.c.), whose structural diagram is shown in Fig. 1. Here **i.e.** is an inertial element; **e.e.** is a nonlinear element having an extremal dependence $y = f(x)$ of the output y on the input x; **s.f.** is a signal-forming device; **s.r.** is a sign relay; **a.o.** is an actuator; x is the input to the e.e.; y is the output of the inertial element. Thus, a system is considered in which the inertia acts after the e.e., a case very frequently encountered and difficult for control.

Fig. 1

The task of e.c. is to find and maintain the quantity y at the maximum value that can be attained at the maximum of the function $f(x)$. However, in practice, because the displacement must be carried out with sufficient speed, the dynamic quantity y, owing to the inertia of the system, will differ from the static value y_{stat}. Therefore the search process will last rather long and will be accompanied by considerable "hunting." Moreover, in the case of large inertia, since the speed of the a.o. must be small, the search time increases still more.

The found value of the dynamic maximum may differ substantially from the sought value of the "potential" maximum ([1–11]).

An increase in the order of the system and its oscillatory character make it difficult to find the extremum in the operation of any e.c. system, both with memorization of the maximum and with superposition of modulating oscillations. It need hardly be said that ordinary e.c. systems cannot be applied to unstable objects.

2. Let us consider a method of extremal control that makes it possible, in principle, to eliminate completely the influence of inertia on the time of search for the extremum, and also to get rid of the unfavorable influence of low-frequency external disturbances. When this method is used, e.c. can be applied both to stable and to unstable systems.

If the dynamic part of the object is nonlinear, then, for a broad class of systems, the differential equation relating the output y and the input x can be written in the form

$$\psi_1(y', y'', \dots, y^{(n)}) + \psi_2(y) = f(x(t)), \tag{1}$$

where $\psi_1 = y^{(n)} + \psi_{11}(y', y'', \dots, y^{(n-1)})$.

Here we shall assume that ψ_{11} and ψ_2 are, generally speaking, discontinuous nonlinear functions of their arguments satisfying conditions

Lipschitz in $y, y', \dots, y^{(n-1)}$; f is a continuous function having an extremal character and attaining an extremum at $x = x^*$, for example, $f(x) = -x^2$.

If the actuator (a.) is not switched on, then $x =$ const. When the actuator is switched on, x changes; if, in particular, its velocity $\dot{x} = k$ is constant in magnitude, then $x = x_0 \pm kt$.

Let us write equation (1) in the form

$$\psi_1(y',\, y'', \dots, y^{(n)}) = f(x) - \psi_2(y). \tag{1'}$$

Suppose that the velocity $\dot{x}$ is sufficiently large and, consequently, the shifting time is small. Then, by virtue of the conditions imposed on the functions ψ_1, ψ_2, and f, the changes in the quantities y and $\psi_2(y)$ will be sufficiently small, tending to zero as the shifting speed of the actuator increases without bound.

We must find the static (“potential”) maximum of the quantity y. It is obvious that it can be attained only at the maximum value of the right-hand side of equation (1), i.e., when $f(x) = f(x)_{\max}$. But the maximum of the function $f(x)$ is attained at the value $x = x^*$.

It is seen from equation $(1')$ that, for $\psi_2(y) =$ const, the maximum of $f(x)$ will correspond to the maximum of the left-hand side, i.e., to the maximum of the function $\psi_1 = \psi_1(y', y'', \dots, y^{(n)})$.

Thus, if the function $\psi_1 = \psi_1(y', \dots, y^{(n)})$ is introduced into the extremal controller and its maximum is sought under a sufficiently rapid shifting of the actuator, then this maximum will be attained at a value $x \simeq x^*$, which makes the function $f(x)$ a maximum. At this, the value x^* is attained after the completion of transients in the inertial element, and the value y is maximized.

The indicated conclusion is exact if $y =$ const, which will occur at an infinitely large shifting speed. In practice this speed is always limited; however, if it is sufficiently large, then finding the maximum of ψ_1 corresponds, with sufficient approximation, to finding $f(x)_{\max}$.

Moreover, as in the case of applying A. P. Yurkevich' s dynamic converter $(5, 6)$, low-frequency disturbances will not pass through the sign relay and will have no effect on the search process.

Especially interesting here is the circumstance that the determination of the extremum will be the more accurate, the greater the shifting speed; meanwhile, in ordinary extremal-control systems, in the presence of an inertial plant, a low shifting speed is necessary, otherwise there will be a large loss due to "overshooting" and a large amplitude of oscillations.

Consider, for example, a first-order plant; in contrast to the usual method of extremal control by the first derivative, in which its zero is sought, it is expedient to carry out a rapid shifting and to seek the maximum of the first derivative.

3. The method described has the following drawback, inherent in any method of derivative control: when the characteristic is flat, the derivatives y', y'', ⋯, etc. are small, since the actuator speed cannot be excessively large. Since there is always a dead zone δ in the sign relay, when the input signal ψ_1 is smaller than the dead zone, the system will not track the extremum either during motion of the actuator relative to the extremum or when the extremum itself is displaced by the action of external disturbances.

To eliminate this drawback, it is expedient to feed to the sign relay the signal

$$\varphi = \psi_1(y', y'', \ldots, y^{(n)}) + \psi_0,$$

where $\psi_0 = \psi_0(\psi_2)$ or $\psi_0 = \psi_0(y)$ is a special nonlinear function of its argument. This function, for the case $\psi_0 = \psi_0(\psi_2)$, has the form shown ⋯

in Fig. 2. Here ψ_2 may be reckoned from the value of its extremum or from the upper (lower) boundary. From Fig. 2 it is seen that ψ_0 increases in proportion to ψ_2 until, at $\psi_2 = \psi_{2\delta}$, it becomes larger than the zone δ. Then, for $\psi_2 > \psi_{2\delta}$, ψ_0 is made an approximately constant quantity. When the function ψ_0 is introduced into the output signal, it may be expedient to modify the form of the function ψ_1.

The transformed form ψ_1^* of the dependence ψ_1 is shown in Fig. 3; until the quantity ψ_0 has passed through the insensitivity zone δ, i.e., while $\psi_1 < \psi_{1\delta}$, $\psi_1^* = 0$. For $\psi_0 > \delta$, the quantity ψ_1^* increases from zero together with ψ_1.

Let us see what positive features the described method of extremal control has:

a) Because the quantity φ includes the nonlinear function ψ_0 of the parameter y, the quantity φ applied to the signum relay will change as if y did not change slowly, i.e., however small the quantities $y', y'', \ldots, y^{(n)}$ may be. Consequently, the extremal-control system will function even with a very flat form of the extremal dependence and with a very slow displacement of the extremum.

Fig. 2

Figure 2: Fig. 2

Fig. 3

Figure 3: Fig. 3

Fig. 2

Fig. 3

b) After the actuator is switched on, because of the form of the function ψ_2, the influence of y ceases to make itself felt, and the subsequent motion is determined by the signal $\psi_1(y', y'', \ldots, y^{(n)})$. Owing to this, the fastest approach to the extremal point x^* with little hunting is ensured, and the influence of low-frequency external disturbances is eliminated.

c) Since the reasoning given above imposes no conditions on the stability character of the control system, it may be concluded that the method set forth is also applicable to neutral and unstable systems. This remarkable feature of the new method can greatly facilitate the realization of an extremal-control system.

Thus, for example, in an undamped oscillatory plant described by the equation

$$a_2 y'' + y = f(x),$$

as well as in an unstable plant of the form

$$a_2 y'' - a_1 y' + y = f(x),$$

application of the described method of extremal control will make it possible quickly to find and maintain the sought values x^* corresponding to the maximum of the right-hand side. At the same time, for existing extremal-control systems considerable damping is desirable or necessary.

The indicated method is also applicable to extremal-control systems with input modulation. In particular, for a first-order plant it is expedient, in contrast to the usual method, instead of slowly displacing the actuator, to displace it rapidly and, in the case of searching for a maximum, to seek not the minimum value of y, but its maximum.

The described method of extremal control is approximately correct in the sense that, when it is used, the maximum of the function ψ_1 will correspond to the true maximum only at an infinitely high switching speed.

Let us now consider a modification of the extremal-control method set forth, which gives a solution of the problem of finding the optimizing quantity x^* at any speed-

the shift. The desired quantity $y_{\text{stat}}^{\max}$ is unknown. However, it will be reached after a practically sufficiently long interval of time if the input x is maintained at such a value $x = x^*$ for which $f(x) = f(x)_{\max}$. But, considering expression (1), we see that the quantity $f(x)$ is exactly equal to the left-hand side of equation (1), regardless of the rate at which the input x is changed. Consequently, if the quantity $\varphi_1 = \psi_1 + \psi_2$ is introduced into the signum relay, then at any rate of shifting of the actuator the extremal controller will register the value x^* corresponding to the extremum of the output y_{stat}, as soon as the input x, in the search process, passes through this value. After this, reversals of the actuator will occur near the value $x = x^*$, or else stopping of the actuator may be provided, while the output quantity y will approach the maximum value at the greatest rate ([3,4]). It was shown ([1–4]) that monotonic displacement of the input is practically unacceptable, since in the presence of external disturbances false operations and loss of the extremum may occur, and that this danger can be avoided by introducing a commutator. It was also shown ([3,4]) that introducing a variable speed of the commutator will allow the system to approach the extremum if monotonic external disturbances also act. All these devices are applicable also when the present variant is used; however, under intense external disturbances the rate of approach to the extremum will be small. In this latter case it is apparently more expedient to use the first variant of the described method, i.e., to introduce the functions ψ_1 or φ into the signum relay.

We have considered an extremal-control system in which the inertial element acts after the extremal element. If there is also an inertial element before the extremal element, then the following way of accelerating the process of searching for the extremum is possible: the position of the actuator is changed, and in doing so the coordinate x of the input is also changed. The function ψ_1, φ_1, or φ is introduced into the signum relay.

The signum relay records the value x^* corresponding to the instant of passage through the maximum of the functions ψ_1, φ, or φ_1.

Thereupon the further search for the extremum is stopped, but an automatic position controller is switched on, controlling the actuator in such a way as to ensure the fastest approach of the quantity x to x^*.

The considerations set forth here are also valid in the case of searching for an extremum in several variables. Application of these considerations substantially accelerates the search for inertial objects when using the Gauss–Seidel methods, the steepest descent method, and others.

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Note: Figure translations are in progress. See original paper for figures.