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Abstract

Full Text

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CONSTRUCTION OF THE GREEN TENSOR FOR THE PROBLEM OF EQUILIBRIUM OF A SHALLOW SHELL OF DOUBLE CURVATURE

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1. As is known ([1,3]), by means of N. A. Kil' chevskii' s method the system of functional equations for a shallow shell rectangular in plan, in displacements, under arbitrary boundary conditions, is represented in the form*

$$u_{(i)\beta}(M,N) = v_{(i)\beta}(M,N) -$$

$$- \int_0^a \int_0^b K^j_{(i\beta)}(Q,M)\, u_{(i)j}(Q,N)\, dx_Q dy_Q + A_{(i)\beta}(M,N). \qquad (1)$$

Using the arbitrariness of the regular part of the auxiliary displacements, we choose it so that the operators $A_{(i)\beta}(M,N)$ vanish. For this purpose the auxiliary displacements $v^*_{(i)\beta}$ must satisfy the conditions of fastening of the shell contour.

It may be verified that, when the shell contour is rigidly fastened, the desired displacements $v^*_{(i)\beta}$ may be given the form**

$$v^*_{(i)\beta}(M,N) = v_{(i)\beta}(M,N) + \sum_{m,n} z^\beta_{mn}(M)\, Z^i_{mn}(N); \qquad (2)$$

$$v_{(i)\beta}(M,N) = \sum_{m,n} A^{(i)\beta}_{mn}\, Z^\beta_{mn}(M)\, Z^i_{mn}(N), \qquad (3)$$

where

$$Z^1_{mn}(M) = \cos\frac{m\pi x_M}{a}\sin\frac{n\pi y_M}{b}, \qquad Z^2_{mn}(M) = \sin\frac{m\pi x_M}{a}\cos\frac{n\pi y_M}{b},$$

$$Z^3_{mn}(M) = \sin\frac{m\pi x_M}{a}\sin\frac{n\pi y_M}{b};$$

$$A_{mn}^{(1)1} = \frac{4\varepsilon}{\pi^2 Eh}\frac{\gamma_{mn}}{\omega_{mn}^2}, \qquad A_{mn}^{(2)2} = \frac{4\varepsilon}{\pi^2 Eh}\frac{\delta_{mn}}{\omega_{mn}^2}, \qquad A_{mn}^{(3)3} = \frac{48(1-\nu^2)\varepsilon a^2}{\pi^4 Eh^3}\frac{1}{\omega_{mn}^2},$$

$$A_{mn}^{(1)2} = A_{mn}^{(2)1} = -\frac{4(1+\nu)^2\varepsilon^2}{\pi^2 Eh}\frac{mn}{\omega_{mn}^2}, \qquad A_{mn}^{(\alpha)3} = A_{mn}^{(3)\alpha} = 0;$$

$$\gamma_{mn} = (1-\nu^2)m^2 + 2(1+\nu)\varepsilon^2 n^2, \qquad \omega_{mn} = m^2 + \varepsilon^2 n^2,$$

$$\delta_{mn} = 2(1+\nu)m^2 + (1-\nu^2)\varepsilon^2 n^2, \qquad \varepsilon = \frac{a}{b};$$

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* Here and below $i, j, \beta, \gamma = 1, 2, 3;\ \alpha = 1, 2;\ m, n, k, l = 1, 2, \ldots, \infty$.

** It can be shown that the operations performed subsequently on the series (2) and (3) are valid in accordance with the theory of generalized functions.

$$z_{mn}^1(M) = \frac{\beta_m x_M - a}{a}\sin\frac{n\pi y_M}{a}, \qquad z_{mn}^2(M) = \frac{\beta_n y_M - b}{b}\sin\frac{m\pi x_M}{a},$$

$$z_{mn}^3(M) = \psi_m(x_M)\left[\psi_n - (y_M) - \sin\frac{n\pi y_M}{b}\right] - \psi_n(y_M)\sin\frac{m\pi x_M}{a};$$

$$\psi_m(x_M) = \frac{m\pi}{a^3}x_M(x_M - a)(\alpha_m x_M - a),$$

$$\psi_n(y_M) = \frac{n\pi}{b^3}y_M(y_M - b)(\alpha_n y_M - b),$$

$$\alpha_k = 1 + (-1)^k, \qquad \beta_k = 1 - (-1)^k.$$

On the basis of what has been said, system (1) is transformed into the system of integral equations

$$u_{(i)\beta}(M,N) = v^*_{(i)\beta}(M,N) - \int_0^a\int_0^b K^{*j}_{(\beta)}(Q,M)u_{(i)j}(Q,N)\,dx_{Qdy}Q, \qquad (4)$$

whose kernels can be represented in the form

$$K^{*j}_{(\beta)}(Q,M) = \sum_{m,n} f_{mn}^{(\beta)j}(Q)Z_{mn}^{\beta}(M). \qquad (5)$$

We do not write out the expressions for the functions $f_{mn}^{(\beta)j}(Q)$ because of their unwieldiness.

2. We seek the solution of system (4) in the form

$$u_{(i)\beta}(M,N) = \sum_{m,n} \left[E_{mn}^{(i)\beta}(N) Z_{mn}^{\beta}(M) + A_{mn}^{(i)\beta} Z_{mn}^{i}(N) z_{mn}^{\beta}(M) \right], \tag{6}$$

where the functions $E_{mn}^{(i)\beta}(N)$ are to be determined.

Substituting (6) into (4) and introducing the notation

$$R_{mnkl}^{(\beta)j} = \int_0^a \int_0^b f_{mn}^{(\beta)j}(Q) Z_{kl}^{j}(Q)\, dx_{Qdy} Q, \qquad T_{mnkl}^{(\beta)j} = \int_0^a \int_0^b f_{mn}^{(\beta)j} z_{kl}^{j}(Q)\, dx_{Qdy} Q,$$

we obtain the following infinite system of equations for the unknown functions:

$$E_{mn}^{(i)\beta}(N) + \sum_j \sum_{k,l} R_{mnkl}^{(\beta)j} E_{mn}^{(i)\beta}(N) = A_{mn}^{(i)\beta} Z_{mn}^{i}(N) - \sum_j \sum_{k,l} A_{kl}^{(i)j} T_{mnkl}^{(\beta)j} Z_{mn}^{i}(N). \tag{7}$$

3. In the case of hinged fastening of the shell contour, one should take

$$v_{(i)\beta}^{*} = v_{(i)\beta}, \qquad K_{(\beta)}^{*j}(Q,M) = K_{(\beta)}^{j}(Q,M) = \sum_{m,n} B_{mn}^{(\beta)j} Z_{mn}^{j}(Q) Z_{mn}^{\beta}(M), \tag{8}$$

where

$$B_{mn}^{(\alpha)1} = B_{mn}^{(\alpha)2} = 0, \qquad B_{mn}^{(1)3} = -\frac{4\varepsilon k_1}{\pi a} \frac{m\alpha_{mn}}{\omega_{mn}^2}, \qquad B_{mn}^{(2)3} = -\frac{4\varepsilon^2 k_1}{\pi a} \frac{n\beta_{mn}}{\omega_{mn}^2},$$

$$B_{mn}^{(3)1} = -\frac{48\varepsilon\rho_1 k_1 a}{\pi^3 h^2} \frac{m}{\omega_{mn}^2}, \qquad B_{mn}^{(3)2} = -\frac{48\varepsilon^2\rho_1 k_1 a}{\pi^3 h^2} \frac{n}{\omega_{mn}^2}, \qquad B_{mn}^{(3)3} = \frac{48\varepsilon\rho_3 k_1 a}{\pi^4 h^2} \frac{1}{\omega_{mn}^2},$$

$$\alpha_{mn} = \rho_1 m^2 - (\chi - \nu - 2)\varepsilon^2 n^2, \qquad \beta_{mn} = [1 - (\nu + 2)\chi] m^2 - \rho_2 \varepsilon^2 n^2,$$

$$\rho_1 = 1 + \nu\chi, \qquad \rho_2 = \nu + \chi, \qquad \rho_3 = 1 + 2\nu\chi + \chi^2, \qquad \chi = \frac{k_2}{k_1}.$$

In this case the solution of the problem is considerably simplified, and the components of the Green tensor are represented in the form

$$u_{(i)\beta}(M,N)=\sum_{m,n}D_{mn}^{(i)\beta}Z_{mn}^{(\beta)}(N)Z_{mn}^{i}(M),\tag{9}$$

where the coefficients $D_{mn}^{(i)\beta}$ are determined from the system of equations

$$D_{mn}^{(i)\beta}+\frac{a^2}{4\varepsilon}\sum_{j}B_{mn}^{(\beta)j}D_{mn}^{(i)j}=A_{mn}^{(i)\beta}.\tag{10}$$

As a result of the calculations we obtain

$$D_{mn}^{(1)1}=\frac{4\varepsilon}{\pi^2Eh}\frac{1}{\omega_{mn}^2}\left(\gamma_{mn}+\frac{Cm^2\alpha_{mn}^2}{\Omega_{mn}}\right),\qquad D_{mn}^{(2)2}=\frac{4\varepsilon}{\pi^2Eh}\frac{1}{\omega_{mn}^2}\left(\delta_{mn}+\frac{C\varepsilon^2n^2\beta_{mn}^2}{\Omega_{mn}}\right),$$

$$D_{mn}^{(3)3}=\frac{4\varepsilon}{\pi^2Eh}\frac{C}{k_1^2a^2}\frac{1}{\omega_{mn}^2}\left(1-\frac{C\theta_{mn}^2}{\Omega_{mn}}\right),$$

$$D_{mn}^{(1)2}=D_{mn}^{(2)1}=-\frac{4\varepsilon^2}{\pi^2Eh}\frac{mn}{\omega_{mn}^2}\left[(1+\nu)^2+\frac{C\alpha_{mn}\beta_{mn}}{\Omega_{mn}}\right],$$

$$D_{mn}^{(1)3}=D_{mn}^{(3)1}=\frac{4\varepsilon C}{\pi Ehk_1a}\frac{m\alpha_{mn}}{\Omega_{mn}},\qquad D_{mn}^{(2)3}=D_{mn}^{(3)2}=-\frac{4\varepsilon^2C}{\pi Ehk_1a}\frac{n\beta_{mn}}{\Omega_{mn}},$$

where

$$\Omega_{mn}=\omega_{mn}^4+C\theta_{mn}^2,\qquad \theta_{mn}=\varkappa m^2+\varepsilon^2n^2,\qquad C=\frac{12(1-\nu^2)a^4k_1^2}{\pi^4h^2}.$$

The expression for the deflection of a shell under the action of a normal concentrated unit force, found by V. Z. Vlasov ([2]), and the expressions for the components of displacement caused by the action of a tangential concentrated unit force, found by M. Mishonov ([4]), coincide respectively with the components of the Green tensor $u_{(3)3}$ and $u_{(1)1}$, $u_{(1)2}$, $u_{(1)3}$, determined by formulas (9).

The results obtained indicate, in particular, the equivalence of the systems of integral and differential equations of shell equilibrium.

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Note: Figure translations are in progress. See original paper for figures.