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Abstract

Full Text

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On Impurity Conductivity of Germanium at Low Temperatures

At a temperature $T \ll \varepsilon_i/k$, where ε_i is the ionization energy of the impurity and k is Boltzmann' s constant, the electrical conductivity of a semiconductor is negligibly small, since in a weak electric field the atoms of the excess impurity remain almost all neutral. However, as the field strength is increased, impact ionization increases, owing to the fact that the mean free paths of the charge carriers are relatively large at low temperatures, while the ionization energy of the impurity is very small and amounts, for example, in the germanium samples doped with indium that we studied, to only 0.01 eV. Therefore impact ionization begins in fields that are unusually weak for this process, at a strength of several volts per centimeter ([1–3]).

Typical curves for the dependence of j, the current density, on E, the field strength, for germanium samples of p-type are shown in Fig. 1. The concentration of the excess acceptor impurity was of the order of 10^{14} cm^{-3}. The lower the temperature, the smaller the role played by thermal ionization and the more clearly one can observe the influence of impact ionization, as is evident from the curves $j = \varphi(E)$ corresponding to different temperatures.

Hall-effect measurements, carried out with direct current in weak fields and with pulses of duration 3 and 100 μsec in stronger fields, showed that the concentration of holes at field strengths $E > E_{\text{br}}$ first increases very steeply, and then more slowly, and at approximately $E \approx 4E_{\text{br}}$ reaches its maximum possible value, equal to $N_a - N_d$, where N_a is the concentration of acceptors and N_d that of donors; E_{br} is the breakdown field, equal for our samples to 5 V/cm. Thus, in the range $E_{\text{br}} \ll E \ll 4E_{\text{br}}$, the concentration of ionized acceptors increases approximately from N_d to N_a.

At the temperature of liquid helium, thermal ionization may be neglected in comparison with impact ionization. In this case the concentration of holes is

$$p = \frac{s(N_a - N_d) - rN_d}{r + s}, \tag{1}$$

where s is the average probability of ionization and r the average probability of recombination.

Fig. 1. Dependence of the current density j on the electric-field strength E, at temperatures 4.2, 14, and 20.4° K

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Fig. 2. Dependence of the drift velocity of holes v on the electric-field strength E at 4.2 and 78° K

Figure 2: Fig. 2. Dependence of the drift velocity of holes v on the electric-field strength E at 4.2 and 78° K

Using the explicit form of the electron distribution function $f(\varepsilon) = \text{const}\exp[-(\varepsilon/\bar{\varepsilon})^n]$ ($\bar{\varepsilon}$, to within a numerical factor, is equal to the mean electron energy), one can show that

$$s = W_i\left(\frac{\bar{\varepsilon}}{\varepsilon_i}\right)^{2n-3/2} e^{-(\varepsilon_i/\bar{\varepsilon})^n}, \qquad \text{where} \quad W_i = \text{const} \sim \pi a_0^2\sqrt{\frac{\varepsilon_i}{m^*}}; \tag{2}$$

a_0 is the radius of the orbit of the bound hole; ε_i is the ionization energy; m^* is the effective mass. The parameter n is equal to 2 if the distribution of electrons is determined by their interaction with acoustic phonons; $n = 1$, if

collisions of electrons with optical phonons or with one another play an important role ([4]). The dependence of the mean energy $\bar{\varepsilon}$ on the field E can be determined from the condition of equality between the energy acquired from the field and that given up to the lattice ([5,6]), i.e.

$$\frac{(eE)^2\tau(\bar{\varepsilon})}{2m^*} = \frac{1}{\tau_{ph}(\bar{\varepsilon})}\frac{\hbar\omega_{\bar{q}}}{2N_{\bar{q}}+1} \simeq \frac{1}{\tau_{ph}(\bar{\varepsilon})}\frac{(\hbar\omega_{\bar{q}})^2}{2kT}, \tag{3}$$

where τ_{ph}^{-1} is the frequency of collisions with the lattice; $\tau(\bar{\varepsilon})$ is the total mean free time; $\hbar\omega_{\bar{q}}$ and $\bar{q}$ are the mean energy and momentum of the phonons with which an electron of energy $\bar{\varepsilon}$ interacts ($\bar{q} = \sqrt{2m^*\bar{\varepsilon}}$, $N_{\bar{q}}$ is the occupation number of such phonons. Here it is assumed that $2N_{\bar{q}} \gg 1$. The reverse assumption does not change the qualitative results of the calculation.

The recombination probability r changes with increasing $\bar{\varepsilon}$ much more slowly than s. Therefore the increase in the concentration of free holes in the prebreakdown region of fields is determined mainly by the exponential increase in the ionization rate.

Fig. 1. Dependence of the current density j on the electric-field strength E, at temperatures 4.2, 14, and 20.4° K

Fig. 2. Dependence of the drift velocity of holes v on the electric-field strength E at 4.2 and 78° K

If the number of charged centers is very small, then the principal scattering mechanism is interaction with acoustic phonons and, consequently, $n = 2$, $\hbar\omega_{\bar{q}} = c\bar{q}$, $\tau(\bar{\varepsilon}) = \tau_{ph}(\bar{\varepsilon}) \sim \bar{\varepsilon}^{-1/2}$, where c is the speed of sound; then $\bar{\varepsilon} \sim E$. In the opposite case ($N_x \gtrsim 10^{13}$), $\tau(\bar{\varepsilon})$ is determined by scattering on charged impurities. Then $\tau(\bar{\varepsilon}) \sim \bar{\varepsilon}^{3/2}$, and the mean energy, and together with it the mobility, hardly change up to fields

$$E_c \sim (N_d/p_0)^{1/2}\sqrt{\delta}\,\varepsilon_i/el_{ph},$$

and then $\bar{\varepsilon}$ increases rapidly ([6]). In the last formula $p_0 = (\pi a_0^2 l_{ph})^{-1}$, $l_{ph} = \tau_{ph}(\bar{\varepsilon})\sqrt{2\varepsilon/m^*}$ is the "phonon" mean free path, $\delta = m^* c^2/kT$. Together with $\bar{\varepsilon}$, the ionization rate will also increase very rapidly.

Having measured the current density j and the hole concentration p, one can find the drift velocity and its dependence on the field strength E. The results of the calculations are shown in Fig. 2 for temperatures 4.2 and 78° K. In the latter case $v = \mu E$ and the mobility $\mu = \text{const}$ up to $E \sim 100$ V/cm. However, at $T = 4.2°$ K the drift velocity depends on E in a complicated way. In weak fields, for $E < E_{\text{pr}}$, the drift velocity $v = \mu_1 E$, where $\mu_1 = \text{const}$. In the dia-

in the range $E_{\text{br}} \ll E \ll 4E_{\text{br}}$ the drift velocity $v = \text{const}$, or the mobility μ is approximately inversely proportional to E; for $E > 4E_{\text{br}}$ (up to $E \sim 50$ V/cm) the drift velocity $v = \mu_2 E$, where again $\mu_2 = \text{const}$. The ratio $\mu_1/\mu_2 = 3.5$.

The sharp drop in mobility at helium temperatures in the range of fields greater than the breakdown field is apparently associated with the appearance, in the bulk of the semiconductor, of a large number of charged centers. In this case the contribution of Coulomb scattering to the total number of collisions per unit time increases substantially,

$$\tau^{-1}(\bar{\varepsilon}) = \tau_{ph}^{-1}(\bar{\varepsilon}) + \tau_c^{-1}(\bar{\varepsilon}), \tag{4}$$

$$\tau_c^{-1}(\bar{\varepsilon}) \sim p(\bar{\varepsilon})\pi a_0^2\sqrt{\bar{\varepsilon}/m^*}\,(\varepsilon_i/\bar{\varepsilon})^2$$

and the distribution function becomes Maxwellian because of electron-electron collisions. Equation (3), taking (4) into account, can be brought to the form

$$\left[\frac{eEl_{ph}}{\varepsilon_i\sqrt{2\delta}}\right]^2 = \left(\frac{\bar{\varepsilon}}{\varepsilon_i}\right)^2 + \frac{p(\bar{\varepsilon})}{p_0}. \tag{5}$$

Fig. 3. Dependence of the drift velocity $\varphi = v\sqrt{2m^*/\delta\varepsilon_i}$ and of the hole concentration $\eta = \dfrac{p(\varepsilon)}{N_a - N_d}$ on the field $\xi = \dfrac{el_{ph}}{2\sqrt{\delta\varepsilon_i}}E$ for different values of the parameter α.

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If the parameter

$$\alpha = \frac{N_a}{p_0}\frac{W_i}{r} \gtrsim 1,$$

then, in fields in which the ionization becomes sufficiently large ($\bar{\varepsilon}$ is then several times smaller than ε_i), the second term on the right-hand side of Eq. (5) begins to play an essential role. In this case $p(\bar{\varepsilon})$, with increasing E, grows no longer exponentially but quadratically, and the mean energy changes very slowly.

The drift velocity

$$v = \sqrt{\frac{2}{m^*}\frac{\delta}{el_{ph}}\frac{\bar{\varepsilon}^{3/2}}{E}} \tag{6}$$

in this range of fields either decreases or remains almost constant, depending on the value of α. Typical curves $\eta(E) = \dfrac{p}{N_a - N_d}$ and $v(E)$ are given in Fig. 3. In the calculation it was assumed that $n = 1$.

After practically all the impurities have been ionized, the role of Coulomb scattering drops sharply because of the rapid decrease of the cross section with increasing energy. The further change in mobility is again determined only by interaction with phonons, but now optical phonons must also be taken into account.

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Note: Figure translations are in progress. See original paper for figures.