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Abstract

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INTERACTION OF ELECTRONS WITH THE FIELD OF THE H_{01} WAVE IN A CIRCULAR WAVEGUIDE

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In connection with the problem of generating millimeter waves, it is of interest to investigate the interaction of an electron beam with the field of an unretarded wave in a waveguide. This question was considered by A. V. Gaponov ([1]), with the assumption that the deviations of the electron trajectories from the equilibrium ones are small and that the equations of motion can be linearized. In ([1]) it was noted that another mechanism of energy transfer by electrons may also occur, associated with nonlinear effects. Such a mechanism is discussed in the present communication for the example of the interaction of a helical beam with the field of the H_{01} wave in a circular waveguide.

Let the wave propagate along the z-axis, coinciding with the axis of the waveguide, and let the static magnetic field have components $(0, 0, H_0)$. An electron enters the waveguide at the point $(r_0, \varphi_0, 0)$ at the instant $t = 0$. The field is determined by the φ-component of the vector potential:

$$A_\varphi = \frac{1}{2} H_0 r - BgJ_1(gr)\sin(hz - \omega t + \alpha); \tag{1}$$

Bg is the amplitude of the high-frequency potential; h is the propagation constant of the (real) wave; g is the transverse wave number; ω is the frequency; α is the initial phase; J is the Bessel function of the first order.

Introduce the dimensionless variables

$$x = gr; \qquad y = gz; \qquad \tau = \Omega t;$$

$$P_x = gP_r/m\Omega; \qquad P_y = gP_z/m\Omega; \qquad \beta = -P_\varphi g^2/2m\Omega,$$

where $\Omega = eH_0/mc$ is the cyclotron frequency; P_r, P_φ, P_z are the corresponding generalized momenta; $\beta = \text{const}$, since the potential does not depend on φ. The dimensionless Hamiltonian of the problem has the form

$$H = \frac{P_y^2}{2} + \frac{P_x^2}{2} + \frac{x^2}{8} - \frac{2\beta^2}{x^2} - \beta + \varepsilon f(x) \sin\left(\frac{h}{g} y - \frac{\omega}{\Omega}\tau + \alpha\right) + \dots; \qquad (2)$$

$$f(x) = (2\beta/x - x/2) J_1(x); \qquad \varepsilon = Bg^2/H_0$$

is a small parameter of the problem (Bg^2 is the amplitude of the z-component of the high-frequency field). The term with ε^2 is omitted, since it gives no contribution in the first approximation of perturbation theory.

There is an exact integral of the motion

$$H_1 = H - \frac{\omega g}{\Omega h} P_y = \text{const}, \qquad (3)$$

which is easily obtained by passing to new variables u, P_u with the aid of the generating function

$$F = xP'_x + \left(y - \frac{\omega g}{\Omega h}\tau + \frac{g}{h}\alpha\right) P_u.$$

The solution of the unperturbed system ($\varepsilon = 0$) has the form:

$$x = 2\{W + \beta + \sqrt{W(W+2\beta)} \sin\psi\}^{1/2}, \qquad (4)$$

$$\dot{x} = \frac{\partial x}{\partial \psi} = \frac{\sqrt{W(W+2\beta)}\cos\psi}{\{W + \beta + \sqrt{W(W+2\beta)}\sin\psi\}^{1/2}};$$

$$P_y = \text{const}; \qquad \psi = \tau + \text{const}. \qquad (4')$$

Here $W = P_x^2/2 + x^2/8 + 2\beta^2/x^2 - \beta = \text{const}$ is the transverse kinetic energy, equal to $\frac{1}{2}x_c^2$; x_c is the dimensionless cyclotron radius of the electron orbit;

$$\beta = \frac{X_o^2 - x_c^2}{4}; \qquad (5)$$

X_o is the dimensionless distance from the waveguide axis to the center of the orbit.

For $\varepsilon \neq 0$, in accordance with (2), we pass to the new variables W, ψ by formulas (4). The equations of motion in the new variables are obtained by differentiating the Hamiltonian H_1. Next, we expand $f[x(W,\psi)]$ in a Fourier series in ψ:

$$f = \sum_{k=0}^{\infty} a_k(W) \sin k\psi + \sum_{k=0}^{\infty} b_k(W) \cos k\psi \qquad (6)$$

and consider resonance at the first harmonic. Using the averaging method (3) and taking into account that $b_1(W) = 0$, we obtain the equations of the first approximation in the form:

$$\dot{W} = -\frac{\varepsilon}{2} a_1(W)\sin\theta; \qquad \dot{P}_u = -\frac{\varepsilon}{2}\frac{h}{g} a_1(W)\sin\theta; \tag{7}$$

$$\dot{\theta} = 1 + \frac{h}{g}P_u - \frac{\omega}{\Omega} - \frac{\varepsilon}{2}\frac{da_1}{dW}\cos\theta; \qquad \theta = \psi + \frac{h}{g}u,$$

where

$$a_1(W) = \frac{1}{\pi}\int_0^{2\pi} f[x(W,\psi)]\sin\psi\, d\psi.$$

From the first and second equations (7) it follows that

$$W - \frac{g}{h}P_u = W_0 - \frac{g}{h}P_u^0. \tag{8}$$

We obtain another integral of system (7) by averaging (3):

$$\frac{P_u^2}{2} - \frac{\omega g}{\Omega h}P_u + W - \frac{\varepsilon}{2}a_1(W)\cos\theta = \frac{P_u^{0\,2}}{2} - \frac{\omega g}{\Omega h}P_u^0 + W_0 - \frac{\varepsilon}{2}a_1(W_0)\cos\theta_0. \tag{9}$$

The investigation shows that the greatest transfer of energy takes place in the case of a strongly off-axis beam. In this case $X_o \gg x_c$, $W/2\beta \ll 1$, and, expanding in powers of $\sqrt{W/2\beta}$, we obtain

$$a_1(W) \simeq C\sqrt{2W}; \qquad C = -\sqrt{\beta}\,[J_2(2\sqrt{\beta}) + J_0(2\sqrt{\beta})]; \tag{10}$$

$$\dot{W} = \left\{\frac{\varepsilon^2 C^2}{2}(W - W_0) + \frac{\varepsilon^2 C^2}{2}W_0 - \left[\frac{h^2}{2g^2}(W - W_0)^2 + \Delta_0(W - W_0) + \frac{\varepsilon C}{2}\sqrt{2W_0}\cos\theta_0\right]^2\right\}^{1/2}. \tag{11}$$

$$\Delta_0 = 1 \pm \frac{h}{g}P_u^0 - \frac{\omega}{\Omega}$$

is the initial detuning. The sign before the radical is determined by the value of $\dot{W}|_{\tau=0}$.

Solving (11), one can, with the aid of (8) and (9), find the loss of the electron' s total kinetic energy ΔE during the transit time,

$$\Delta E = \frac{\omega}{\Omega}\Delta W.$$

If the relative energy transfer $(W - W_0)/W_0$ is small, then for the energy transfer averaged over the initial phases we have

$$\overline{W - W_0} = -\frac{g^2}{h^2}\Delta_0 + \frac{g^2}{h^2}\frac{\Delta_0}{2\pi}\int_0^{2\pi} \frac{\operatorname{dn}\left(\frac{h^2 b\tau}{2g^2},\, k\right) d\theta_0}{1 - k^2\cos^2\frac{\theta_0}{2}\,\operatorname{sn}^2\left(\frac{h^2 b\tau}{2g^2},\, k\right)}, \tag{12}$$

where

$$b^2 = \frac{g^4}{h^4}\Delta_0^2 + \left|\frac{2eg^2}{h^2}C\sqrt{2W_0}\right|\cos^2\frac{\theta_0}{2}; \qquad k = \frac{\left|\frac{2eg^2}{h^2}C\sqrt{2W_0}\right|}{b^2}.$$

dn, sn are Jacobi elliptic functions.

The energy loss depends substantially on the magnitude and sign of the initial detuning. For $\Delta_0 = 0$, $\overline{W - W_0} = 0$. If $\Delta_0 > 0$, then $\overline{W - W_0} < 0$, i.e., the unbunched electrons on average give up energy. If, however, $\Delta_0 < 0$, then $\overline{W - W_0} > 0$.

The case

$$\frac{g^4\Delta_0^2}{h^4} \gg \left|\frac{2eg^2}{h^2}C\sqrt{2W_0}\right|$$

corresponds to the linear regime considered in (1). The optimal detuning is

$$\Delta_0 \sim \frac{h^2}{g^2}\left|\frac{2eg^2}{h^2}C\sqrt{2W_0}\right|^{1/2}.$$

In this case the energy lost can be estimated by numerical integration of expression (12). Taking, for example, the waveguide radius $a \sim 3$ cm, $\omega = 6\cdot 10^{10}$ s^{-1}, $g/h = 1$, $\Delta_0^2 = 5\cdot 10^{-8}$,

$$\left|\frac{2eg^2}{h^2}C\sqrt{2W_0}\right| = 10^{-7},$$

the transverse velocity $v_0 = 2\cdot 10^9$ cm/s, the longitudinal velocity $v_z^0 = 3.5\cdot 10^8$ cm/s, and the transit length $L = 120$ cm, we find that

$$\overline{W - W_0}/W_0 \sim 16\%.$$

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Note: Figure translations are in progress. See original paper for figures.