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Abstract

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ON THE QUESTION OF THE INFLUENCE OF BOTTOM RELIEF ON THE ANTARCTIC CIRCUMPOLAR CURRENT

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To explain the cause of bends in the streamlines of the Antarctic circumpolar current, in work ([1]) the following equation was proposed:

$$\frac{\sqrt{\omega A_z \cos\theta}}{H^2}\Delta\Psi + \frac{\partial\Psi}{\partial\theta}\frac{1}{\sin\theta}\frac{\partial}{\partial\lambda}\left(\frac{2\omega\cos\theta}{H}\right) - \frac{1}{\sin\theta}\frac{\partial\Psi}{\partial\lambda}\frac{\partial}{\partial\theta}\left(\frac{2\omega\cos\theta}{H}\right) =$$

$$= \frac{a_0}{\sin\theta}\frac{\partial}{\partial\theta}\left[\frac{\tau_\lambda\sin\theta}{H}\right]. \quad (1)$$

Here Ψ is the function of total transports; λ is longitude ($0 \leqslant \lambda < 2\pi$); $\theta = \pi/2 + \varphi$, where φ is the latitude of the place; ω is the angular velocity of the Earth' s rotation, equal to $7.29 \cdot 10^{-5}$ sec^{-1}; H is depth; a_0 is the mean radius of the Earth, equal to $6.37 \cdot 10^8$ cm; $\tau_\lambda = \tau'_\lambda/\rho_0$; τ_λ is the zonal component of wind stress; ρ_0 is the mean density; A_z is the coefficient of vertical exchange; Δ is the Laplace operator.

By estimating the order of magnitude of the terms entering equation (1), qualitative conclusions were obtained about the connection between bends of isolines of the function Ψ and large-scale irregularities of the bottom ([1]). In the present work a method is given for solving equation (1) for a prescribed transport of the current Q and wind field $\tau_\lambda(\theta)$.

First of all, the question arises of what is to be understood by the function H entering equation (1). We are studying the large-scale features of the circumpolar current (the characteristic horizontal scale is of the order of 1000 km); we are not interested in small-scale features. Therefore it is natural that by the function H one should understand not the true values of the depth at each point, but a smoothed bottom relief.

Let us represent the true depth H in the form

$$H = H^* + H',$$

where H^* is the large-scale component of H; H' is random small-scale disturbances superposed on H^*. The smoothing of the bottom relief H must be carried out so as to isolate the component H^* in the best possible way.

We proceed as follows. Since H^* contains no small-scale components, in the region $40^\circ S \leqslant \varphi \leqslant 70^\circ S,\ 0 \leqslant \lambda < 2\pi$ one may represent H^* in the form

$$H^* = \sum_{n=0}^{n_0} \sum_{m=0}^{m_0} \left\{ \cos 6m \left(\varphi - \frac{2\pi}{9} \right) [a_{mn} \cos n\lambda + b_{mn} \sin n\lambda] \right\}. \tag{2}$$

The number of harmonics in expression (2) is determined by the scale of those features of the bottom relief which we wish to represent in H^*. Since the compo-

the component H' can be regarded as a random variable; the coefficients a_{mn} and b_{mn} can be determined from the condition

$$\sum_{k=1}^{k_0} [H_k - H_k^*]^2 = \min; \tag{3}$$

the summation is carried out over the points where the values of H are known.

Calculations were made for the following data: $k_0 = 540$, $n_0 = 4$, $m_0 = 2$. These calculations made it possible to determine the general character of H^*, and on their basis, in accordance with the features of the bottom relief in the Antarctic Ring, isolines were drawn and the values of the function $\cos\theta/H^*$ were computed.

Introduce the function $\xi = \xi_0 \dfrac{\cos\theta}{H^*}$, where ξ_0 is a certain scale. Consider those isolines ξ that encircle the continent of Antarctica and are closed curves. Let D be the region made up of these isolines. In the region D, $|\operatorname{grad}\xi| \neq 0$ and ξ varies from ξ_1 to ξ_2.

It can be shown that the streamlines of the Antarctic circumpolar current are located in the region D and are sufficiently close to the isolines of the function ξ. Therefore we shall seek a solution of equation (1) in the region D with the boundary conditions $\Psi|_{\xi_1} = 0$, $\Psi|_{\xi_2} = Q$.

For convenience in the calculations it is necessary to pass from the surface of the sphere, on which equation (1) is considered, to the plane of the map by means of the method indicated in ([2]). If one uses a map made in the direct stereographic projection, then equation (1) becomes

$$\frac{\sqrt{\omega A_z N}}{H^{*2}} \Delta\Psi + \frac{2\omega}{\xi_0} \left[\frac{\partial\Psi}{\partial r} \frac{\partial\xi}{r\,\partial\lambda_1} - \frac{\partial\Psi}{r\,\partial\lambda_1} \frac{\partial\xi}{\partial r} \right] = \frac{a_0}{qr} \frac{\partial}{\partial r} \left[\frac{r\tau_\lambda(r)}{MH^*} \right]. \tag{4}$$

Here r, λ_1 are polar coordinates on the map plane; their relation to the coordinates θ, λ is: $r = 2q\tan\dfrac{\theta}{2}$, $\lambda = \lambda_1$; the number q is determined by the scale of the map; $M(r) = 1 + (r/2q)^2$; $N(r) = [1 - (r/2q)^2]/M$.

In the region D, a grid was formed from the coordinate lines r and λ_1. The step of this grid on the map plane was chosen so that $\delta r \simeq r_0\delta\lambda \simeq 2$ cm (this corresponds approximately to 250 km). At the nodes of this grid the coefficients of equation (4)* were calculated. The scales Q_0 and ξ_0 (Q_0 and ξ_0 are the scales for ψ and ξ, respectively) can be chosen in such a way that the derivatives of ψ and ξ will be of order 1. Then, after introducing the scale Q_0, equation (4) takes the form

$$\varepsilon A(r,\lambda_1)\Delta\Psi_1 + \frac{\partial\Psi_1}{\partial r}\frac{\partial\xi}{r\,\partial\lambda_1} - \frac{\partial\Psi_1}{r\,\partial\lambda_1}\frac{\partial\xi}{\partial r} = \varepsilon F(r,\lambda_1). \tag{5}$$

The functions $A(r,\lambda_1)$ and $F(r,\lambda_1)$ are of order 1; $A(r,\lambda_1) > 0$. The parameter $\varepsilon = 0.6 \cdot 10^{-2}$. Boundary conditions: $\Psi_1|_{\xi_1} = 0$, $\Psi_1|_{\xi_2} = 1$. Equation (5) is an equation with the small parameter ε.

Since ε stands at the Laplace operator, a solution by the usual finite-difference method is hardly possible because of the necessity of choosing a very fine grid. It is therefore natural to seek a solution of equation (5) in the form of an expansion in an asymptotic series in ε. We shall confine ourselves to finding the principal term of such an expansion. Denote it by Ψ_0.

Let us pass to a new orthogonal coordinate system ξ, η; $\xi = \xi(r,\lambda_1)$, $\eta = \eta(r,\lambda_1)$. This coordinate system is naturally associated with equation (5). The family of isolines of the function η is found from the function ξ by the condition of orthogonality. Equation (5) in the coordinates ξ, η becomes

$$\varepsilon A\left\{\frac{\partial^2\Psi_1}{\partial\xi^2}\frac{|\,\mathrm{grad}\,\xi|^2}{(\xi,\eta)} + \frac{\partial^2\Psi_1}{\partial\eta^2}\frac{|\,\mathrm{grad}\,\eta|^2}{(\xi,\eta)} + \frac{\partial\Psi_1}{\partial\xi}\frac{\Delta\xi}{(\xi,\eta)} + \frac{\partial\Psi_1}{\partial\eta}\frac{\Delta\eta}{(\xi,\eta)}\right\} - \frac{\partial\Psi_1}{\partial\eta} = \varepsilon\frac{F}{(\xi,\eta)}; \tag{6}$$

* The values of the function $\tau_\lambda(r)$ were communicated to the author by Yu. A. Ivanov.

(ξ,η) is the notation for the Jacobian of the functions ξ and η. The domain D in the new coordinates is transformed into the ring $\xi_1 \leqslant \xi \leqslant \xi_2$, $0 \leqslant \eta < \eta_0$. The boundary conditions for Ψ_1 are: $\Psi_1|_{\xi_1} = 0$; $\Psi_1|_{\xi_2} = 1$.

We now proceed to the determination of the function Ψ_0. If, for some component of the solution of equation (6) (the equation is linear), the derivative with respect to η plays an essential role, then this component must be of order ε (otherwise the equation will not be satisfied, since the term $\partial\Psi_1/\partial\eta$ cannot be balanced by the other terms, which are of order ε). Therefore, in order to find Ψ_0, it is necessary to extract from the solution Ψ_1 the maximal part depending only on ξ (in this case the term with the derivative with respect to η vanishes). Thus, $\Psi_0 = \Psi_0(\xi)$. The function $\Psi_0(\xi)$ can be found from the equation

$$A\,\overline{\frac{|\operatorname{grad}\xi|^2}{(\xi,\eta)}}\,\frac{d^2\Psi_0}{d\xi^2} + A\,\overline{\frac{\Delta\xi}{(\xi,\eta)}}\,\frac{d\Psi_0}{d\xi} = \overline{\frac{F}{(\xi,\eta)}} \tag{7}$$

under the conditions $\Psi_0(\xi_1) = 0$, $\Psi_0(\xi_2) = 1$, where the notation has been introduced

$$\overline{f(\xi,\eta)} = f_0(\xi) = \frac{1}{\eta_0}\int_0^{\eta_0} f(\xi,\eta)\,d\eta.$$

From equation (7) the function $\Psi_0(\xi)$ can be easily found.

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CITED LITERATURE

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2. I. A. Kibel' , *Introduction to the Hydrodynamic Methods of Short-Range Weather Forecasting*, 1957.

Note: Figure translations are in progress. See original paper for figures.