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# PENETRATION OF POINTED AXISYMMETRIC BODIES INTO SOILS

![Fig. 1. Typical curves of dynamic compression of loam of air-dry
moisture content](figure)

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Fig. 1. Typical curves of dynamic compression of loam of air-dry moisture content

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## Abstract

## Full Text

## CONTINUUM MECHANICS

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# PENETRATION OF POINTED AXISYMMETRIC BODIES INTO SOILS

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On the basis of the results of experiments carried out at Moscow State University for soils of low moisture content, Kh. A. Rakhmatulin introduced a "plastic gas" as the modeling medium. This is a continuum which, under loading, changes its density according to a definite law, but upon unloading preserves the density attained. Repeated loading does not change the density until the stress reaches the original maximum value. Figure 1 shows the dynamic compression curve obtained by N. A. Alekseev for a specimen of loam placed in a cylindrical rigid tube. Along the vertical axis is plotted the pressure $P = -\sigma_r$, acting on the end face of the specimen in the direction of the tube axis; along the horizontal axis, the deformation. The duration of the loading and unloading process in these experiments is of the order of 1/500 sec. When loading the specimen, for example, with cylindrical symmetry, the property of a "plastic gas" is preserved, but the dependence of the normal stress $\sigma_r$ on deformation may be different.

**Fig. 1.** Typical curves of dynamic compression of loam of air-dry moisture content

At present there are no experimental data for such cases of dynamic loading. Therefore, in solving the problem posed, it was assumed that the compression curve obtained also applies under cylindrical loading. In those cases where tangential stresses play a secondary role, this assumption apparently will not introduce a significant error. In the general case it seems more natural to assume that the relation between the deformation and the mean normal stress

$$\frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3) = f(1 - \rho_0/\rho)$$

does not depend on the manner of loading.

In the experiments mentioned above, even under strong compressions the temperature of the soil increased only slightly, and no noticeable influence of the

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rate of loading on the character of the dynamic compression was detected. On this basis it is assumed that the dynamic compression curves are also applicable at the shock front formed in the plastic gas. Finally, in order to determine the stressed state of the plastic gas, let us assume that the condition of limiting equilibrium is satisfied not only at the instant when equilibrium is lost, but also during the subsequent motion of the soil.

As experiments carried out at Moscow State University have shown, when cylinders with a conical nose having an apex angle up to 30° penetrate into soil, no noticeable change in the shape of the free surface is ob–

is observed. In this case the craters formed behind the penetrating body had a cylindrical shape. This is also confirmed by experiments on the penetration of solid bodies into various plastic media, the results of which are given in ([1]).

On the basis of the above, the problem of the penetration of a pointed axisymmetric body into a plastic gas was solved under the following assumptions: throughout the entire motion the particles of the medium remain in planes parallel to the free surface; in each such plane the motion is propagated by a shock wave, and the density of the medium changes only at the shock wave. Below are the results of solving the problem of the vertical penetration of a cylindrical body with a conical nose. Under the assumptions made, the problem reduces to the investigation of motion with cylindrical symmetry.

The equations of motion and conservation of mass in Lagrangian variables are written in the form

$$\rho_0 r \frac{\partial^2 u}{\partial t^2} = (r+u)\frac{\partial \sigma_r}{\partial r} + (\sigma_r - \sigma_\theta)\frac{\partial}{\partial r}(r+u), \tag{1}$$

$$\frac{1}{2}\frac{\partial}{\partial r}(r+u) = \frac{\rho_0}{\rho}\, r. \tag{2}$$

The condition of limiting equilibrium (the Prandtl plasticity condition) for the case under consideration takes the form

$$\sigma_\theta - \sigma_r = \tau_0 \pm \mu(\sigma_\theta + \sigma_r); \qquad \tau_0 = 2k\cos\vartheta; \qquad \mu = \sin\vartheta, \tag{3}$$

where $k$ is the cohesion coefficient, and $\vartheta$ is the angle of internal friction.

Substituting the value of $\sigma_\theta$ from (3) into equation (1), we obtain

$$(r+u)\frac{\partial \sigma_r}{\partial r} + \nu \frac{\partial}{\partial r}(r+u)\sigma_r = \rho_0 r \frac{\partial^2 u}{\partial t^2} + \frac{\tau_0}{1 \mp \mu}\frac{\partial}{\partial r}(r+u); \tag{4}$$

$$\nu = \mp \frac{2\mu}{1 \mp \mu}. \tag{5}$$

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In these formulas the minus sign corresponds to the plus sign in condition (3), and conversely.

The region of perturbed motion will be an inhomogeneous incompressible medium enclosed between the circumference of the shock wave and the circumference that is the line of intersection of the penetrating body with the plane of the section under consideration. This region may be divided into $n$ bands, whose boundaries are contact surfaces with a density discontinuity. As penetration proceeds, the number of bands in the given section will increase. In this approximate formulation the penetration problem was solved. The calculations showed that, with high accuracy, the acceleration can be determined at constant density behind the shock wave. This density is determined from the dynamic compression curve and the pressure at the shock wave, determined through the initial velocity of the penetrating body. Under this condition, the solution of equations (4) and (2) on the conical nose of the body gives

$$-\sigma_r = \ddot{H}\frac{\rho_0}{\nu b}\left(a^{\nu/2}-1\right)\operatorname{tg}^2\beta x + \dot{H}^2\frac{\rho_0}{b(\nu-2)}\left[\frac{\nu-2}{\nu}\left(a^{\nu/2}-1\right)+b(\nu-2)a^{\nu/2}-\right.$$

$$\left.-\left(a^{\nu/2}-1\right)\right]\operatorname{tg}^2\beta + P_0a^{\nu/2} + \left(a^{\nu/2}-1\right)\frac{\tau_0}{\nu(1\mp\mu)}; \qquad 0\le x\le H, \tag{6}$$

where $\beta$ is half the cone aperture angle; $H$ is the current penetration depth; $\dot{H}$ and $\ddot{H}$ are the velocity and acceleration of the penetrating body; $b=\rho_0/\rho$; $\rho_0$ is the initial density of the soil; $\rho$ is the density behind the shock wave; $a=(1-b)^{-1}$; $P_0$ is the initial pressure in the soil.

The action of the soil on the body occurs over the expanding nose part up to the point of separation, which is determined in the course of solving the problem. The absence of axial deformation causes an axial stress $\sigma_z$. For nor-

normal and tangential components of the stress acting on an area adjoining the generatrix of the conical nose, we shall have

$$\sigma_n = \sigma_r\cos^2\beta + \sigma_z\sin^2\beta; \qquad \sigma_\tau = (\sigma_z-\sigma_r)\sin\beta\cos\beta, \tag{7}$$

where $n$ is the outward normal; $\tau$ is directed along the generatrix from the tip of the body. If the penetrating body is perfectly smooth, then $\sigma_\tau$ must be equal to zero and

$$\sigma_n = \sigma_r = \sigma_z. \tag{8}$$

This equality, together with condition (3), expresses the state of complete plasticity ([2]). Under this assumption, for the acceleration of the penetrating body we shall have ([3])

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$$-\ddot{H} = \frac{\alpha}{2}\,\frac{(v_0^2 + c/\alpha)H^2}{[1+\omega H^3]^{\alpha/3\omega+1}}, \tag{9}$$

where

$$\omega = \frac{2\pi\rho_0\,\mathrm{tg}^4\,\beta\,(a^{\nu/2}-1)}{3mvb}; \qquad c = \frac{2\pi\,\mathrm{tg}^2\,\beta}{m}\left[P_0 a^{\nu/2} + (a^{\nu/2}-1)\frac{\tau_0}{1+\mu}\right];$$

$$\frac{\alpha}{\omega} = 3\frac{\nu}{\nu-2}\,\frac{((\nu-2)/\nu)(a^{\nu/2}-1) + b(\nu-2)a^{\nu/2} - (a^{\nu/2}-1)}{a^{\nu/2}-1}. \tag{10}$$

Here $m$ is the mass of the penetrating body; $v_0$ is its initial velocity. In the initial period of penetration, when the velocity of the body is still high, internal friction and cohesion may be neglected (an ideal plastic gas). In this case formulas (10) give

$$\omega = \pi\rho_0\,\mathrm{tg}^4\,\beta\ln a/3mb;$$

$$\alpha/\omega = 3(1 + b/\ln a); \qquad c = 0.$$

The velocity of the penetrating body is determined by the formula

$$\dot{H}^2 = \frac{v_0^2 + c/\alpha}{[1+\omega H^3]^{\alpha/3\omega}} - \frac{c}{\alpha}. \tag{11}$$

**Fig. 2. Acceleration curves.**
$I$ $-\mu_0 = 0,\ \tau_0 = 0,\ \nu = 0;$
$II$ $-\mu_0 = 0.3,\ \tau_0 = 9396\ \mathrm{kg/m^2},\ \nu = 0;$
$III$ $-\mu_0 = 0.3,\ \tau_0 = 9396\ \mathrm{kg/m^2},\ \nu = 0.50971$

Let sliding friction, proportional to the normal stress, take place between the soil and the surface of the body. Then, according to (7), $\sigma_z$ is determined from the equality

$$\mu_0(\sigma_r\cos^2\beta + \sigma_z\sin^2\beta) = (\sigma_z - \sigma_r)\sin\beta\cos\beta.$$

The coefficient of friction $\mu_0$ is assumed constant. Introduce the notation

$$\chi = \frac{1+\mu_0\,\mathrm{ctg}\,\beta}{1-\mu_0\,\mathrm{tg}\,\beta}.$$

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Fig. 3. Curves of the dependence of the velocity of a penetrating body on depth. $\mu_0 = 0.3$, $\tau_0 = 9396$ kg/m$^2$, $\nu = 0.50971$

Figure 2: Fig. 3. Curves of the dependence of the velocity of a penetrating body on depth. $\mu_0 = 0.3$, $\tau_0 = 9396$ kg/m$^2$, $\nu = 0.50971$

In the presence of friction, the acceleration and velocity will be obtained from formulas (9) and (11), if in them the quantities $\alpha, \omega, c$ are multiplied by $\chi$. These formulas are valid up to the depth $H \leq h$, where $h$ is the height of the conical nose. For $H > h$, the formula for the acceleration takes the form

$$\ddot{H} = \ddot{H}_h \exp\left[-\frac{\alpha h^2}{1+\omega h^3}(H-h)\right]. \tag{12}$$

Here $\ddot{H}_h$ is the acceleration at the depth $H = h$, calculated by formula (9). The solution presented is valid for both signs of condition (3), but for a prescribed compressive stress $\sigma_r$, the presence of internal friction will apparently lead to the appearance of a certain tensile force in the tangential-

cial direction. The latter will reduce the compressive stress $\sigma_\theta$ that would occur in the absence of internal friction. It therefore seems more natural to assume that $\sigma_r$, in absolute value, is greater than $\sigma_\theta$; then in condition (3) the minus sign must be taken. Under this assumption we carried out calculations[3].

Figure 2 gives curves of acceleration as a function of depth for different soil characteristics. Allowing for the variability of the density behind the shock wave leads to cumbersome formulas that determine the law of penetration. Calculations using these formulas were carried out on the "Strela" computer at Moscow State University. Figure 3 gives curves of the dependence of the velocity on depth for constant (1) and variable (2) density behind the shock wave. The curves show a substantial influence of the change in density on the magnitude of the velocity and on the depth of penetration.

Fig. 3. Curves of the dependence of the velocity of a penetrating body on depth. $\mu_0 = 0.3$, $\tau_0 = 9396$ kg/m$^2$, $\nu = 0.50971$

Investigation of the problem made it possible to obtain an approximate, but simple, formula for determining the velocity at any depth. Up to the depth $H \leq h$, the velocity is determined with great accuracy by formula (11). At the depth $h \leq H \leq 2h$, the velocity is given by the formula

$$\dot{H}^2 = \left(\dot{H}_h^2 + \frac{c}{\alpha}\right)\exp\left[-\frac{\alpha h^2}{1+\omega h^3}(H-h)\right] - \frac{c}{\alpha}. \tag{13}$$

The velocity $\dot{H}_h$ is obtained from (11) for $H = h$. In expression (13), $\alpha$, $\omega$, and $c$ are determined from formulas (10), but in them $b = \rho_0/\rho_{\mathrm{av}}$, where $\rho_{\mathrm{av}}$

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is the arithmetic mean of the densities determined behind the shock wave by means of the compression curve and the pressures calculated at the initial body velocity $v_0$ and at the velocity $\dot{H}_h$. For the velocity of the body at a depth $(n-1)h \leq H \leq nh$, the approximate formula for the velocity has the form

$$\dot{H}^2 = \left(\dot{H}^2_{(n-1)h} + \frac{c}{\alpha}\right) \exp\left[-\frac{\alpha h^2}{1+\omega h^3}(z-h)\right] - \frac{c}{\alpha}, \qquad h \leq z \leq 2h. \qquad (14)$$

Here $\dot{H}_{(n-1)h}$ is the velocity at the depth $H = (n-1)h$, which is calculated by a formula analogous to (14), written for the preceding interval. The quantities $\alpha$, $\omega$, and $c$ are likewise determined from formula (11), but $\rho_{\text{av}}$ in the expression $b = \rho_0/\rho_{\text{av}}$ is the arithmetic mean density behind the shock wave, determined with the aid of the pressures on it at the penetration velocities $\dot{H}_{(n-2)h}$, $\dot{H}_{(n-1)h}$, and the dynamic compression curve. $\dot{H}_{(n-2)h}$ is the velocity at the depth $H = (n-2)h$. This process is continued until the body comes to a complete stop, $\dot{H}(H_m) = 0$; $H_m$ will then be the depth of penetration. Calculations showed that the depth determined by the method presented here differs from the depth determined on a computer using the more cumbersome formulas by no more than 3%.

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## References

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*Note: Figure translations are in progress. See original paper for figures.*

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