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Abstract

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Interaction of a Simple Wave with a Contact Discontinuity

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The problem considered is an element of many practical problems of one-dimensional gas flow. Numerical computation is possible by the method of characteristics or by the method of finite differences (^{1}).

Below an exact solution of the problem is given for a discrete series of values of the adiabatic exponent κ, defined in the form

$$\kappa = \frac{2n+3}{2n+1}, \quad n \text{ integer.} \tag{1}$$

The scheme of the flow regions when a simple wave passes through a contact discontinuity is given in Fig. 1.

In general, 4 cases are possible (a combination of a rarefaction or compression wave and a discontinuity in which the speed of sound on the right is greater $T_>$ or less $T_<$). Shown here is a rarefaction wave passing through the discontinuity $T_>$.

In regions I, III, and IV there are simple waves, respectively incident, reflected, and transmitted. In region II there is a general unsteady flow, representing the interaction of the incident simple wave with the wave reflected from the contact discontinuity.

For the discrete values of κ (or n) under consideration, the flow in region II is determined by a function $\chi(c, v)$ (^{2}) of the form

$$\chi(c,v) = \left(\frac{\partial}{c\,\partial c}\right)^{n-1} \left[\frac{1}{c}\,\varphi\left(c + \frac{\kappa-1}{2}v\right) + \frac{1}{c}\,\psi\left(c - \frac{\kappa-1}{2}v\right)\right], \tag{2}$$

where v is the velocity of the gas particles; c is the speed of sound; φ and ψ are arbitrary functions.

The coordinates of the flow plane are expressed through $\chi(c, v)$ in the form

$$t = \frac{\kappa-1}{2c}\frac{\partial\chi}{\partial c}, \quad x = vt - \frac{\partial\chi}{\partial v}. \tag{3}$$

Fig. 1

Figure 1: Fig. 1

The functions φ and ψ in the problem under consideration are determined by the boundary conditions on the characteristic AB and the segment AC of the contact discontinuity. The condition on AB will be (^{2})

$$\chi(c, v) = -\int f(v)\, dv \quad \text{for} \quad c - \frac{\kappa - 1}{2} v = c_0. \tag{4}$$

Here $f(v)$ is a function characterizing the incident simple wave:

$$x = (v + c)t + f(v);$$

c_0 is the speed of sound in the undisturbed gas to the left of the contact discontinuity.

This condition gives an ordinary linear differential equation of order $(n-1)$ with a right-hand side for the function φ. Condition (4) has been studied in solved problems on the interaction of simple waves (^{3}), reflection of a simple wave from a wall, and from a free boundary. It has been shown

(3), that the general solution of the homogeneous equation corresponding to (4) may be set equal to zero, since it represents $\varphi(u)$ in the form of a polynomial in u with coefficients proportional to $b_1, b_2, \ldots, b_{n-1}$, $\psi(c_0), \psi'(c_0), \ldots, \psi^{(n-1)}(c_0)$ (where $b_1, b_2, \ldots, b_{n-1}$ are constants of the general solution), and all these constants are immaterial, since they disappear in the expression $\chi(c, v)$.

Thus, condition (4) determines the function $\varphi(u)$ as a particular solution of the nonhomogeneous linear equation following from (4), if one sets

$$\psi(c_0) = \psi'(c_0) = \ldots = \psi^{(n-1)}(c_0) = 0. \tag{5}$$

The condition at the contact discontinuity is obtained from the following considerations:

1. The discontinuity does not move relative to the gas particles, i.e., on it

$$\frac{dx}{dt} = v. \tag{6}$$

Fig. 1

2. The flow to the right of the discontinuity is a simple wave, in which

$$c_2 - \frac{\varkappa - 1}{2} v = c_{20} \tag{7}$$

(c_2 is the speed of sound to the right of the discontinuity, c_{20} is the speed of sound in the undisturbed gas on the right).

3. The ratio of the densities ρ_1/ρ_2 on both sides of the discontinuity does not change when the simple wave passes. This follows from the isentropic nature of the flow on each side of the discontinuity,

$$\frac{\rho_1}{\rho_1'} = \left(\frac{p_1}{p_1'}\right)^{\varkappa}, \qquad \frac{\rho_1}{\rho_1'} = \left(\frac{\rho_2}{\rho_2'}\right)^{\varkappa},$$

whence

$$\frac{\rho_1}{\rho_2} = \frac{\rho_1'}{\rho_2'} = \text{const.}$$

4. The pressure is continuous in passing through the contact discontinuity. It follows from this that

$$\frac{c}{c_2} = \sqrt{\frac{\varkappa p}{\rho_1}} \Big/ \sqrt{\frac{\varkappa p}{\rho_2}} = \sqrt{\frac{\rho_2}{\rho_1}},$$

and relation (7) may be written on the contact discontinuity for the gas on the left in the form

$$c\sqrt{\frac{\rho_1}{\rho_2}} - \frac{\varkappa - 1}{2} v = c_0 \sqrt{\frac{\rho_1}{\rho_2}}. \tag{7a}$$

Thus, it must be that

$$\frac{dx}{dt} = v \quad \text{along the line on which} \quad c = c_0 + \frac{\varkappa - 1}{2}\sqrt{\frac{\rho_2}{\rho_1}}\, v.$$

Using (3), after simple calculations we obtain the condition at the contact discontinuity for the flow in region II:

$$\frac{\partial^2 \chi}{\partial v^2} + \frac{\varkappa - 1}{2}\sqrt{\frac{\rho_2}{\rho_1}} \frac{\partial^2 \chi}{\partial v\, \partial c} - \frac{\varkappa - 1}{2c} \frac{\partial \chi}{\partial c} = 0 \quad \text{for } c = c_0 + \frac{\varkappa - 1}{2}\sqrt{\frac{\rho_2}{\rho_1}}\, v. \tag{8}$$

This condition gives an ordinary differential Euler equation with a right-hand side for determining the function ψ.

For example, for $\chi = 1.4$ ($n = 2$) and for the case of a centered rarefaction wave ($f(v) = 0$), from condition (4) the function $\varphi(u) \equiv 0$. Then condition (8) for determining $\psi(u)$ gives the equation

$$a^3c^3\xi''' + 3(2-a)a^2c^2\xi'' + 3(5-a)(1-a)ac\xi' + 15(1-a)^2\xi = 0,$$

where $a = \sqrt{\rho_2/\rho_1}$, $\xi = \xi(c)$, $\psi(u) = \xi[(au - c_0)/(1-a)]$.

The constants entering into the solution of the equation following from (8) are determined by conditions (5) and by the condition that the coordinates of point A of the contact discontinuity, determined from (3) (for $c = c_0$, $v = 0$), coincide with the values determined directly from the given incident wave and the initial position of the contact discontinuity (one condition).

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Note: Figure translations are in progress. See original paper for figures.

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