PHYSICS

I. N. MININ

1960

SovietRxiv

Abstract

Full Text

PHYSICS

I. N. MININ

A POINT SOURCE OF LIGHT IN AN ABSORBING MEDIUM BETWEEN PARALLEL PLANES

(Presented by Academician V. A. Ambartsumian, 14 III 1960)

In considering problems on the illumination of surfaces irradiated by light sources, it is necessary to take into account the effect of multiple reflections, which leads to self-illumination of the surfaces. Accounting for this effect reduces to the solution of certain integral equations. One such problem was solved in the work of V. V. Sobolev ([1]), who considered the case of a point source of light located between parallel isotropically reflecting planes. It was assumed there that the space between the planes is transparent.

The present note is devoted to a generalization of V. V. Sobolev' s work by including a homogeneous absorbing medium between the planes. In solving the problem we have used the method proposed by V. V. Sobolev in ([1]).

Let there be a point source of light of luminous intensity I, independent of direction, situated in a medium with absorption coefficient α, bounded by parallel isotropically reflecting planes. We introduce the following notation: H is the distance between the planes; h_1 and h_2 are the distances of the light source from the planes; ρ_1 and ρ_2 are the reflection coefficients of the planes; r is the distance from the perpendicular to the planes drawn through the light source; $E_1(r)$ and $E_2(r)$ are the illuminations of the planes. The quantities $E_1(r)$ and $E_2(r)$ are the unknowns.

Introducing rectangular coordinates on each of the planes, with origins at the points of intersection of the planes with the aforementioned perpendicular, we find

$$E_1(r) = \frac{\rho_2}{\pi} H^2 \int_{-\infty}^{+\infty} dx' \int_{-\infty}^{+\infty} \frac{E_2(r') \exp\left[-\alpha\sqrt{H^2+(x'-x)^2+(y'-y)^2}\right]}{\left[H^2+(x'-x)^2+(y'-y)^2\right]^2}\, dy' + \\ + \frac{Ih_1 \exp\left[-\alpha\sqrt{h_1^2+r^2}\right]}{(h_1^2+r^2)^{3/2}} \quad (1)$$

and an analogous expression for $E_2(r)$. Introducing the optical thickness $\tau_0 = \alpha H$ of the layer situated between the planes, and also putting $I = 1$ and $H = 1$, which corresponds to expressing the illuminations in units of I/H^2, instead of (1) we obtain

$$E_1(r) = \frac{\rho_2}{\pi}\int_{-\infty}^{+\infty} dx' \int_{-\infty}^{+\infty} \frac{E_2(r')\exp\left[-\tau_0\sqrt{1+(x'-x)^2+(y'-y)^2}\right]}{\left[1+(x'-x)^2+(y'-y)^2\right]^2}\,dy' + \\ + \frac{h_1\exp\left[-\tau_0\sqrt{h_1^2+r^2}\right]}{(h_1^2+r^2)^{3/2}}. \tag{2}$$

To solve the obtained system of integral equations, introduce, instead of $E_1(r)$, a new quantity $G_1(x)$ by the relation

$$G_1(x) = \int_{-\infty}^{+\infty} E_1(r)\,dy = 2\int_{|x|}^{\infty} \frac{E_1(r)r}{\sqrt{r^2-x^2}}\,dr, \tag{3}$$

which is Abel' s integral equation. Inverting (3), we find

$$E_1(r) = -\frac{1}{\pi}\int_r^{\infty} \frac{G_1'(x)}{\sqrt{x^2-r^2}}\,dx. \tag{4}$$

Integrating equation (2) with respect to y, we find

$$G_1(x) = \rho_2\int_{-\infty}^{+\infty} A(x'-x)G_2(x')\,dx' + B_1(x), \tag{5}$$

where

$$A(z) = \frac{2}{\pi}\int_0^{\infty} \frac{\exp\left[-\tau_0\sqrt{1+z^2+y^2}\right]}{(1+z^2+y^2)^2}\,dy. \tag{6}$$

$$B_1(z) = 2h_1\int_0^{\infty} \frac{\exp\left[-\tau_0\sqrt{h_1^2+z^2+y^2}\right]}{(h_1^2+z^2+y^2)^{3/2}}\,dy. \tag{7}$$

Introduce the quantities

$$\varphi_1(t) = \int_{-\infty}^{+\infty} G_1(x)e^{-itx}\,dx,$$

$$\overline{A}(t) = \int_{-\infty}^{+\infty} A(x)e^{-itx}\,dx, \qquad \overline{B}_1(t) = \int_{-\infty}^{+\infty} B_1(x)e^{-itx}\,dx. \tag{8}$$

Then from (5) and the analogous expression obtained from the relation for $E_2(r)$ by integration with respect to y, we obtain:

$$\begin{aligned} \varphi_1(t) &= \rho_2\overline{A}(t)\varphi_2(t) + \overline{B}_1(t), \\ \varphi_2(t) &= \rho_1\overline{A}(t)\varphi_1(t) + \overline{B}_2(t). \end{aligned} \tag{9}$$

Note that $B_2(z)$ is obtained from $B_1(z)$ by replacing h_1 with h_2. Solving system (9), we find

$$\varphi_1(t) = \frac{\overline{B}_1(t) + \rho_2\overline{A}(t)\overline{B}_2(t)}{1 - \rho_1\rho_2\overline{A}^2(t)}. \tag{10}$$

Inverting the Fourier integral and applying (4), and also using the known relation

$$\int_r^{\infty} \frac{\sin tx}{\sqrt{x^2 - r^2}}\,dx = \frac{\pi}{2}J_0(rt), \tag{11}$$

where $J_0(x)$ is the Bessel function of order zero, we finally have

$$E_1(r) = \frac{1}{2\pi}\int_0^{\infty} \frac{\overline{B}_1(t) + \rho_2\overline{A}(t)\overline{B}_2(t)}{1 - \rho_1\rho_2\overline{A}^2(t)} J_0(rt)\,t\,dt. \tag{12}$$

Formula (12) gives the exact analytical solution of the problem under consideration.

For computations by formula (12) it is necessary to know the functions $\bar{A}(t)$, $\bar{B}_1(t)$, and $\bar{B}_2(t)$. Using (6), (7), and (8), it is easy to find:

$$\bar{B}_1(t) = 4h_1\tau_0 \int_1^{\infty} \frac{\sqrt{y^2-1}}{y} K_0\left(h_1\sqrt{\tau_0^2 y^2 + t^2}\right) dy, \tag{13}$$

$$\bar{A}(t) = \frac{4}{\pi}\int_1^{\infty} \frac{\sqrt{y^2-1}}{y^3}\left[\sqrt{\tau_0^2 y^2 + t^2}\,\Psi_1\left(\sqrt{\tau_0^2 y^2 + t^2}\right) + \Psi_2\left(\sqrt{\tau_0^2 y^2 + t^2}\right)\right] dy, \tag{14}$$

where $K_0(x)$ is the Macdonald function of order zero,

$$\Psi_n(x) = \int_1^\infty \frac{e^{-tx}}{t^n\sqrt{t^2-1}}\,dt. \tag{15}$$

We note that $\Psi_0(x) = K_0(x)$ and

$$\Psi_n(x) = \int_x^\infty dx \int_x^\infty dx \cdots \int_x^\infty K_0(x)\,dx, \tag{16}$$

where the integration is carried out n times.

In the particular case where the space between the planes is transparent, $\tau_0 = 0$. From (13) and (14), for this case we find

$$\bar{B}_1(t) = 2\pi e^{-h_1 t}, \qquad \bar{A}(t) = t^2 \int_1^\infty e^{-tx}\sqrt{x^2-1}\,dx = tK_1(t), \tag{17}$$

where $K_1(t)$ is the Macdonald function of the first order. Substituting (17) into (12), we obtain the solution of V. V. Sobolev.

Received
11 III 1960

REFERENCES CITED

1. V. V. Sobolev, DAN, **42**, 176 (1944).

Note: Figure translations are in progress. See original paper for figures.