HOMOGENEOUS RAPID DEFORMATION OF TURBULENCE IN A GAS

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Abstract

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V. A. FROST

HOMOGENEOUS RAPID DEFORMATION OF TURBULENCE IN A GAS

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When turbulence passes through shock waves, nozzles, and diffusers, a redistribution of the turbulent and mean energy of the flow takes place. If the change in the flow parameters—pressure, density, and velocity—occurs sufficiently rapidly, so that in considering the change in turbulence it is possible to neglect the forces of viscosity and inertia, then, in order to estimate the character of the change in turbulence, one may apply the condition of constancy of vorticity, and the problem becomes linear. For the case of an incompressible fluid this problem has been solved ([1,2]). In the same formulation, the compressibility of the medium can be taken into account, and the results may possibly be applied to the behavior of turbulence in supersonic flows and in the combustion zone of subsonic flows.

The condition for applicability of the theory of rapid deformation of turbulence is given by the relations between the time of action of the deformation τ_1 and the decay time of the turbulence τ_2. Namely,

$$\tau_1 \ll \tau_2.$$

This condition leads to the following relation between the flow parameters:

$$\frac{\sqrt{\overline{u'^2}}}{U} \ll \frac{l}{D}, \tag{1}$$

where $\sqrt{\overline{u'^2}}$ is the root-mean-square fluctuation velocity, U is the mean flow velocity, l is the scale of turbulence, and D is the length over which the deformation acts (the length of a nozzle, diffuser, or combustion chamber).

Under these assumptions the condition of conservation of vorticity is valid,

$$\omega = \text{const.} \tag{2}$$

Then

$$\omega_i'(x) = \omega_j(a)\frac{\partial x_i}{\partial a_j},$$

where x and a are the Lagrangian coordinates of a fixed volume element, with a before the deformation and x after the deformation. Assuming that the deformation is homogeneous and that before the action of the deformation the flow was uniform and the turbulence homogeneous, we may regard $\partial x_i/\partial a_j$ as constant. Then, from the continuity equation in Lagrangian form, we have:

$$\left|\frac{\partial x_i}{\partial a_j}\right| = \frac{\rho_0}{\rho_1}.$$

Passing to the principal axes of the deformation-velocity tensor, we have:

$$\omega_i'(x) = e_i\omega_i(a), \tag{3}$$

where e_1 are the components of the deformation velocity relative to the principal axes. Using (3), we can relate the spectral turbulence tensors before and after deformation

$$\omega_i(x) = \varepsilon_{ijk}\frac{\partial u_k}{\partial x_j};$$

ε_{ijk} is a tensor equal to 0 if i, j, k are not all different; equal to 1 if i, j, k form the sequence 123123, and equal to -1 if the sequence has the form 132132. Then

$$\varepsilon_{ipq}\frac{\partial u_q'}{\partial x_p} = \varepsilon_{ipq}\frac{\partial x_i}{\partial a_j}\frac{\partial u_q}{\partial a_p}. \tag{4}$$

Taking rot of both sides of (4) and taking into account the constancy of $\partial x_i/\partial a_j$, we have

$$-\Delta u' = \varepsilon_{rsi}\varepsilon_{jpq}\frac{\partial x_i}{\partial a_j}\frac{\partial a_t}{\partial x_s}\frac{\partial^2 u}{\partial a_p \partial a_t}. \tag{5}$$

Introducing the representation of the velocity of the turbulent motion in the form

$$\mathbf{u}'(\mathbf{x}) = \int e^{i\mathbf{x}\chi}\, dz'(\chi), \qquad \mathbf{u}(\mathbf{a}) = \int e^{i\varkappa\mathbf{a}}\, dz(\varkappa) \tag{6}$$

and χ, defined by the relation

Figure 1

Figure 1: Figure 1

$$\chi \cdot \mathbf{x} = \varkappa \cdot \mathbf{a}, \tag{7}$$

we obtain

$$\chi^2 dz'(\chi) = -\varepsilon_{rsi}\varepsilon_{jpq} \frac{\partial x_i}{\partial a_j} \frac{\partial a_t}{\partial x_s} \varkappa_p \varkappa_t \, dz(\varkappa). \tag{8}$$

Using the relation between the spectral energy tensor and the quantity $dz(\varkappa)$, we obtain the connection between the spectral turbulence tensors

$$\Phi'_{11}(\chi) = \frac{E(\varkappa)}{4\pi\chi^4} \left(\chi_2^2 + \chi_3^2\right), \qquad \Phi'_{ii}(\chi) = \frac{E(\varkappa)}{4\pi\chi^4\varkappa^2} \left(\chi^2\varkappa^2 e_2^2/e_1^2 + e_2^2 e_1^2 \chi^4\right). \tag{9}$$

Here we have used the fact that the turbulence of the incident flow is isotropic and, consequently,

$$\Phi_{ij} = \frac{E(\varkappa)}{4\pi\varkappa^4} \left(\varkappa^2 \delta_{ij} - \varkappa_i \varkappa_j\right). \tag{10}$$

In addition, we considered axisymmetric deformation, i.e. $e_2 = e_3$. It is not difficult to obtain analogous formulas for the plane case. Using the relations

$$\overline{u_1^2} = \int \Phi_{11}(\chi)\, d\chi, \qquad \overline{u_2^2} + \overline{u_3^2} = \int (\Phi_{22} + \Phi_{33})\, d\chi, \tag{11}$$

one can obtain the relation between the turbulence intensities in the incident flow and in the section under consideration.

Fig. 1. Change in the square of the longitudinal component $\mu_0 = \mu/c$ and the transverse component $\nu_0 = \gamma/c$ of the turbulent velocity in a cylindrical tube of constant cross-section as a function of the flow deformation

$$c = \sqrt[3]{e_1^2/e_2^2}.$$

The ratio of the intensities of the longitudinal components of velocity will be

$$\bar{\mu} = \frac{\left(V\overline{u_1'^2}/v\right)_1}{\left(V\overline{u_1'^2}/v\right)_0} = \frac{v_0}{v_1} \frac{\left(V\overline{u_1'^2}\right)_1}{\left(V\overline{u_1'^2}\right)_0} = e_2 \sqrt{\mu_0(c)}; \tag{12}$$

similarly, $\bar{\nu} = e_2\sqrt{\nu_0(c)}$, where ν_0 and μ_0 are auxiliary functions of $c = \sqrt[3]{e_1^2/e_2^2}$, shown in Fig. 1 ($\nu_0 = \nu/c$ and $\mu_0 = \mu/c$, where μ and ν in Fig. 1 coincide with the functions given in ([2]) for the case $\rho = \text{const}$).

For $c \geqslant 1$,

$$\nu_0 = \frac{\nu}{c} = \frac{3}{4} + \frac{3}{4c^3}\left[\frac{1}{2\alpha^2} - \frac{1-\alpha^2}{4\alpha^3}\ln\frac{1+\alpha}{1-\alpha}\right],$$

$$\mu_0 = \frac{\mu}{c} = \frac{3}{4c^3}\left[\frac{1+\alpha^2}{2\alpha^3}\ln\frac{1+\alpha}{1-\alpha} - \frac{1}{\alpha^2}\right], \tag{13}$$

where $\alpha = 1 - c^{-3}$.

For $c \leqslant 1$,

$$\mu_0 = \frac{3}{4c^3}\left\{\frac{a^2-1}{a^3}\operatorname{arc\,tg} a + \frac{1}{a^2}\right\},$$

$$\nu_0 = \frac{3}{4} + \frac{3}{4c^3}\left\{\frac{a^2+1}{2a^3}\operatorname{arc\,tg} a - \frac{1}{2a^2}\right\}, \tag{14}$$

where $a = c^{-3} - 1$.

Thus, knowing the ratio of densities in the sections under consideration and the ratio of the mean flow velocities, we can estimate the change in turbulence intensity.

If the continuity equation is written in the form $\rho_0/\rho_1 = e_1e_2^2 = (vF)_1/(vF)_0$, then it is seen that the quantity e_1 can characterize the ratio v_1/v_0, and e_2^2 the ratio F_1/F_0. Then from formulas (13) and (14) we come to the conclusion that, for constant F, acceleration of the flow sharply decreases the longitudinal fluctuating velocity, whereas the transverse component decreases only insignificantly.

It should be noted that in the case of incompressibility of the fluid e_2 and e_1 are not independent. Therefore acceleration of the flow leads to a sharp

Fig. 2. Change in the relative intensity of the transverse component of velocity, measured by the optical-diffusion method during combustion in a cylindrical tube of diameter 100 mm. A—during combustion behind a fine-scale turbulizing grid with holes of diameter 7 mm, as a function of the flow acceleration v_2/v_1. B—during combustion behind a large-scale turbulizing grid with holes of diameter 20 mm, as a function of the flow heating T_2/T_1. 1—calculated curves, 2—experimental data ([4]).

decrease not only of the longitudinal, but also of the transverse component of the turbulent velocity.

These results are in agreement with the measurements of works ([3,4]) and with our measurements. In measuring helium diffusion in a supersonic flow, turbulence intensities varying from 4 to 10% were obtained, depending on the turbulence of the flow in the receiver. The measurements of works

(3), carried out with a hot-wire anemometer, gave values $\varepsilon \simeq 0.1$—0.4%. This difference is explained by the fact that when a hot-wire anemometer is used, the longitudinal fluctuating component of the flow velocity is measured, whereas in the optical-diffusion method it is the transverse component. Table 1 gives the change in the relative intensity of the transverse component of the turbulent velocity downstream of a supersonic nozzle as a function of the Mach number.

Table 1

M	$\varepsilon_{\text{exp.}}$	$\varepsilon_{\text{calc.}}$
0	5.5	5.5
1.56	5.2	5.0
1.88	5.0	4.9
2.14	4.15	4.9
2.36	4.4	4.85

Comparison of these conclusions with experiment was carried out not only for turbulence in supersonic flows, but also for turbulence behind the combustion zone in a cylindrical tube, according to the measurements of work (4) (see Fig. 2).

It should be noted that formulas (13) and (14) can explain the phenomenon only qualitatively, since they were obtained under a large number of assumptions whose admissibility is relative.

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REFERENCES

1. G. I. Taylor, *Zs. angew. Math. u. Mech.*, **15**, 91 (1935).
2. G. I. Taylor, *The Theory of Homogeneous Turbulence*, IL, 1955.
3. L. S. G. Kovaznay, *J. Aeronaut. Sci.*, **20**, 10, Oct. (1953).
4. A. G. Prudnikov, *Measurement of the Turbulence of Air Flows and Flames by the Optical-Diffusion Method*, Dissertation, Moscow, 1956.

Note: Figure translations are in progress. See original paper for figures.