ELECTRICAL ENGINEERING

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Abstract

Full Text

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ON THE NUMBER OF SYMMETRY TYPES OF CONTACT $(1, k)$-POLES

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1. There are in all $Q = C_\mu^k$ functionally distinct oriented contact $(1, k)$-poles realizing a basic sequence [1] of Boolean functions $f_1, f_2, \dots, f_k$ (the numbering is immaterial) of n variables $x_1, x_2, \dots, x_n$, where $\mu = 2^{2^n} - 2$. All $(1, k)$-poles that can be obtained from a given one by transformations from the group O_n of order $2^n n!$ of the hypercube belong to one type. Let $s \in O_n$ induce a permutation S of degree 2^n of the unit constituents σ_i $(i = 0, 1, \dots, 2^n - 1)$, a permutation t of degree μ of the Boolean functions $f \not\equiv \text{const}$ of n variables, and a permutation T of degree Q of the $(1, k)$-poles: $s \to S \to t \to T$.

The set of $2^n n!$ permutation $Q \times Q$-matrices A_T will be a representation Γ of the group O_n, which is reducible and contains the identity representation as many times as there are types of $(1, k)$-poles. Hence the number of types of $(1, k)$-poles is

$$N_{n,k} = \frac{1}{2^n n!} \sum_c n_c \chi(c),$$

where n_c is the number of elements of the class C; $\chi(c)$ is the character of the class C of the group O_n in the representation Γ.

If $(t_1, t_2, \dots, t_\nu)$ is the cycle structure of the permutation t $(\nu < 2^{e^n/e})$, i.e.

$$\sum_{i=1}^{\nu} i t_i = \mu; \qquad \sum_{i=1}^{m} i b_i = k \quad (m = 1, 2, \dots, k), \qquad s \in C,$$

then

$$\chi(c) = \sum \prod C_{t_i}^{b_i};$$

the sum is taken over all decompositions $(b_1, b_2, \dots, b_m)$ of the number k into positive summands, and $C_{t_i}^{b_i}$ is computed only for those i that satisfy the condition $b_i \le t_i$. To determine the cycle structure $(t_1, t_2, \dots, t_\nu)$ we use the fact that the unit constituents stand in the same relation to all functions of logic algebra as the variables do to the constituents. It follows from this that if some cycle $(x_{i_1} x_{i_2} \dots x_{i_p})$ induces a permutation of 2^p constituents of the form $x_{i_1}^{\varepsilon_1} x_{i_2}^{\varepsilon_2} \dots x_{i_p}^{\varepsilon_p}$, with cycle structure $(S_1, S_2, \dots, S_p)$, then the cycle $(\sigma_{i_1} \sigma_{i_2} \dots \sigma_{i_p})$ induces a permutation of all 2^p Boolean functions of the form $\bigvee_{j=1}^{p} \sigma_{i_j}^{\varepsilon_j}$ (where, as usual, $\varepsilon_j = 1$ or 0), whose cycle structure is $(\lambda_1, \lambda_2, \dots, \lambda_p)$, and $\lambda_1 = S_1,\ \lambda_2 = S_2, \dots, \lambda_p = S_p$. To the cycle structure $(\lambda_1, \lambda_2, \dots, \lambda_p)$

we carry the expression

$$P(p) = \sum_{j=1}^{p} \lambda_j \tau_j.$$

Then (as in (2)) the cyclic structure of the substitution t will be characterized by the expression

$$P = \prod_q P(p_q) = \prod_q \left(\sum_{j=1}^{p_q} \lambda_j \tau_j \right) = t_1' \tau_1 + t_2 \tau_2 + \dots + t_\nu \tau_\nu,$$

where

$$t_1' = t_1 + 2, \qquad \sum_q p_q = 2^n, \qquad \tau_{j_1} \tau_{j_2} = j_1 j_2 / j_3 \cdot \tau_{j_3},$$

and j_3 is the least common multiple of the numbers j_1 and j_2.

The computations gave the following results:

n	$k=1$	$k=2$	$k=3$	$k=4$
2	4	19	61	154
3	20	993	62334	3626757
4	400	5883751	122520509746	2001547791980875

For $k = 1$, result (2) is obtained by increasing all numbers of the corresponding graph by 2.

2. Any $(1, k)$-pole with n variables converts n-valued code combinations into k-valued ones. Therefore, for $k < n$, $N_{n,k}$ is also the number of types of transformations resembling the processes for decoding combinations of a redundant code.

3. It is easy to show that

$$N_{n,k} \sim \frac{Q}{2^n n!}.$$

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2. D. Slepian, Canad. J. Math., 5, No. 2, 185 (1953).

Note: Figure translations are in progress. See original paper for figures.