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Abstract

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CALCULATION OF THE DISPLACEMENT OF THE CRYSTALLIZATION FRONT IN A SUPERCOOLED CLOUD UNDER THE ACTION OF SOLID CO_2

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As a first approximation one may assume that the propagation of crystallization in a supercooled cloud under the action of solid carbon dioxide is reduced to the diffusion of ice nuclei, produced under the action of carbon dioxide, and to the transfer of water from droplets to ice crystals by evaporation of the droplets and sublimational growth of the crystals. Proceeding from this representation of the mechanism of crystallization in a supercooled cloud, in our work ([1]) a closed system of equations was obtained that makes it possible to calculate the course of the process in time for the case of one-dimensional propagation:

$$\frac{\partial u}{\partial \tau} = k\frac{\partial^2 u}{\partial x^2} + q_1 + q_2; \qquad \bar{r}_1 = \left\{ R_1^2 - \frac{2D}{\rho_1} \int_0^\tau \left[u(\bar{\eta}_1, \tau') - u_1 \right] d\tau' \right\}^{1/2}; \quad (1)$$

$$\bar{r}_2 = \left\{ \frac{2D}{\rho_2} \int_0^\tau \left[u(\bar{\eta}_2, \tau') - u_2 \right] d\tau' \right\}^{1/2},$$

where u is the concentration of vapor in the cloud; u_1 and u_2 are the values of the vapor concentration corresponding to equilibrium over droplets and crystals at the cloud temperature; τ is time; x is the horizontal coordinate (the OX axis is arranged perpendicular to the vertical plane that is the seeding plane, i.e., we shall assume that at the initial instant ice-crystal nuclei arose in it under the influence of solid CO_2 dropped from an aircraft that flew over the cloud); k is the coefficient of turbulent diffusion; R_1 is the initial value of the radius of droplets in the cloud (at the initial instant the cloud is regarded as monodisperse); ρ_1 and ρ_2 are, respectively, the densities of water and ice; D is the coefficient of molecular diffusion of water vapor.

The quantities q_1 and q_2 are the densities of vapor sources due, respectively, to evaporation of droplets and growth of crystals. For them the following relations were obtained:

$$q_1 = -4\pi D(u - u_1) \int_{x_1(x,\tau)}^{\infty} \bar{r}_1(x, x_1, \tau) \, \frac{n_1}{2\sqrt{\pi k \tau}} \, e^{-(x-x_1)^2/4k\tau} \, dx_1, \qquad (2)$$

$$q_2 = -4\pi D(u - u_2) \, \bar{r}_2 \, \frac{\varepsilon}{2\sqrt{\pi k \tau}} \, e^{-x^2/4k\tau}, \qquad (3)$$

where n_1 is the initial value of the concentration of droplets in the cloud, and ε is the initial value of the number of ice nuclei per 1 cm^2 of the seeding plane. By $x_1(x, \tau)$ here is denoted the solution of the functional equation

$$R_1^2 = \frac{2D}{\rho_1} \int_0^{\tau} \left[u(\bar{\eta}_1, \tau') - u_1 \right] \, d\tau' \qquad (4)$$

with respect to x_1. The quantities $\bar{\eta}_1 = \dfrac{x - x_1}{\tau}\tau'$ and $\bar{\eta}_2 = \dfrac{x}{\tau}\tau'$ denote the paths of droplets and crystals corresponding to the mean radii, respectively, of droplets and crystals $\bar{r}_1(x, x_1, \tau)$ and $\bar{r}_2(x, \tau)$.

As observations show, the process of cloud crystallization occurs mainly in a narrow zone separating the crystallized and uncrystallized parts of the cloud. The most essential role in the vapor balance in the crystallization zone is played by the transfer of vapor from droplets to crystals, which proceeds rather rapidly; diffusion of vapor and droplets to the zone plays a secondary role here and, to a first approximation, may be neglected. System (1) in this case takes the form

$$\frac{\partial u}{\partial \tau} = 4\pi D \left[n_1 r_1 (u_1 - u) - \frac{\varepsilon}{2\sqrt{\pi k \tau}} e^{-x^2/4k\tau} \, \bar{r}_2 (u - u_2) \right]; \qquad (5)$$

$$r_1 = \left\{ R_1^2 - \frac{2D}{\rho_1} \int_0^{\tau} \left[u(x, \tau') - u_1 \right] d\tau' \right\}^{1/2}; \qquad \bar{r}_2 = \left\{ \frac{2D}{\rho_2} \int_0^{\tau} \left[u(\bar{\eta}_2, \tau') - u_2 \right] d\tau' \right\}^{1/2}.$$

As a result of solving system (5), the vapor concentration in the cloud $u(x, \tau)$, the droplet size $r_1(x, \tau)$, and the mean size of the ice crystals $\bar{r}_2(x, \tau)$ can be found.

In carrying out experiments on the action of solid CO_2 on supercooled clouds, the characteristic of the process of cloud crystallization most accessible to measurement is the size of the crystallized zone, recorded at definite time intervals from the beginning of seeding; from it the position of the boundary of the crystallized zone—the crystallization front—is readily found. On the other hand, the position of the front may be determined from the character of the change in vapor concentration $u(x, \tau)$, found as a result of solving system (5). The approach of the crystallization front to any point x corresponds to the moment of a significant decrease in the vapor concentration at that point.

Figure 1. Course of vapor concentration in time at the point $x = 10^5$ cm; the moment $\tau = \tau_*$ is taken as the moment when the crystallization front approaches the given point

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Let us give an example of a numerical solution for $u(x, t)$, in the course of which the position of the front is easily detected. The calculation was carried out for a cloud for which $n_1 = 500$ cm^{-3} and $R_1 = 0.41 \cdot 10^{-3}$ cm (which corresponds to a water content of $0.15 \cdot 10^{-6}$ g $\cdot$ cm^{-3}). The temperature of the cloud was taken to be -15°; therefore, for u_1 and u_2 the values $1.640 \cdot 10^{-6}$ and $1.426 \cdot 10^{-6}$ g $\cdot$ cm^{-3}, respectively, were taken, corresponding to saturation over plane water and ice surfaces. This is justified by the fact that, in the vapor balance, the principal role is played by large droplets and crystals, for which the dependence of vapor concentration on their radius may be neglected. In the calculation the following values of the quantities entering the equation were also adopted: $D = 0.2$ cm$^2 \cdot$ sec^{-1}, $\varepsilon = 10^9$ cm^{-2}, $k = 7.5 \cdot 10^5$ cm$^2 \cdot$ sec^{-1}.

The results of calculating the course of the vapor concentration in time at the point $x = 10^5$ cm, i.e., at a distance of 1 km from the seeding plane, are given in Fig. 1. It is seen that the crystallization front passes through the selected point during the interval from 420 to 520 sec, i.e., it represents a rather narrow region. This feature in the distribution of vapor concentration is in good agreement with observations in nature.

The displacement of the crystallization front, described by the function $u(x, \tau)$, is determined by the values of the quantities entering into it: ε, k, R_1, n_1, u_1, and u_2. Of primary interest is the investigation of the character of propagation of the front in a cloud as a function of the initial concentration of ice nuclei ε, determined by the dosage of solid carbon dioxide seeded from an aircraft, and of the magnitude of the coefficient of turbulent diffusion in the cloud k.

For this purpose we carried out a series of calculations of the displacement of the front, starting from the solution of system (5) by a numerical method on the high-speed electronic computer "Strela." The program was constructed in such a way,

so that, as a result of solving system (5), the times t_* were obtained (see Fig. 1) at which the concentration at subsequent points from 250 to 2000 m (at intervals of 250 m) decreases by $0.002 \cdot 10^{-6}$ g $\cdot$ cm^{-3}. Thus, as a result of solving the system, the position of the crystallization front as a function of time

Fig. 2

Figure 2: Fig. 2

was obtained. The calculation was carried out for 12 variants, corresponding to values ε of 10^7, 10^8, and 10^9 cm^{-2} and values of k of $0.9 \cdot 10^5$, $2.5 \cdot 10^5$, $6.4 \cdot 10^5$, and 10^6 $cm^2 \cdot s^{-1}$. From the positions of the front found in this way, curves of front displacement were constructed for various ε and k. A family of such curves, computed for $k = 2.5 \cdot 10^5$ $cm^2 \cdot s^{-1}$, is shown in Fig. 2. The curves give an idea of the time interval after which the crystallization front reaches a specified distance from the beginning of seeding. Naturally, this effect will be the greater, the larger ε is.

Fig. 2. Curves of displacement of the crystallization front, corresponding to $k = 2.5 \cdot 10^5$ cm^2/s; 1 $-\varepsilon = 10^7$ nuclei/cm^2, 2 $-\varepsilon = 10^8$, 3 $-\varepsilon = 10^9$

To make possible the use of the calculations we have performed, all 12 curves were approximated by the simple formula

$$\tau_* = ax^b. \tag{6}$$

The numerical values of the coefficients a and b, corresponding to x expressed in meters and τ_* in seconds, for various values of ε and k, are given in Table 1.

Table 1

ε	$k = 0.9 \cdot 10^5$	$k = 2.5 \cdot 10^5$	$k = 6.4 \cdot 10^5$	$k = 10^6$
Values of a				
10^7	0.00478	0.00269	0.00151	0.00144
10^8	0.00380	0.00240	0.00102	0.00126
10^9	0.00302	0.00148	0.00203	0.00224
Values of b				
10^7	1.97	1.92	1.88	1.83
10^8	1.97	1.90	1.89	1.81
10^9	1.97	1.94	1.77	1.70

At present there is no reliable information regarding the magnitude ε—the concentration of ice nuclei actually introduced into a cloud when it is treated with solid CO_2, since this quantity has essentially never been measured directly. There is also little information concerning k—the coefficient of turbulent diffusion in a cloud undergoing crystallization. As I. Langmuir pointed out [2], the release of the latent heat of crystallization gives rise to temperature gradients in the cloud and leads to additional turbulence in it. Thus, there is reason to suppose that the magnitude of k in a crystallizing cloud is greater than in clouds in a state of dynamic equilibrium.

Comparison of the displacements of the crystallization front in a supercooled cloud, calculated in the present work, with the displacements observed in experiments on treating clouds with solid CO_2 will make it possible to obtain an indirect estimate of the actual values of ε and k. The cloud parameters adopted here are quite often encountered in stratiform clouds, on which experiments in artificial modification are carried out.

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2. I. Langmuir, in: *Physics of the Formation of Precipitation*, IL, 1951.

Note: Figure translations are in progress. See original paper for figures.