ON THE INTERACTION OF STRONG BLAST WAVES WITH AN ELECTROMAGNETIC FIELD

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Abstract

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ON THE INTERACTION OF STRONG BLAST WAVES WITH AN ELECTROMAGNETIC FIELD

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In the present note we consider the problem of a strong point explosion in an infinitely conducting medium in the presence of a weak magnetic field, and also investigate the question of the interaction of plane shock waves that ionize a gas with a weak electromagnetic field.

1. Let us consider the problem of a strong point explosion ([1]) in a perfect gas with infinitely large electrical conductivity in the presence of a weak homogeneous magnetic field $\mathbf{H}_1$. For definiteness, consider the case in which the explosion energy E_0 is released at a point that we take as the origin of a spherical coordinate system. Taking into account the weakness of the magnetic field, in the first approximation one may neglect its influence on the motion of the medium ([2]). The problem reduces to finding the changes occurring as a result of the explosion in the initial magnetic field. Owing to the homogeneity of the field $\mathbf{H}_1$, the problem thus posed will possess axial symmetry.

In the adopted formulation, the induction equation

$$\frac{\partial \mathbf{H}}{\partial t} = \operatorname{rot}[\mathbf{v} \times \mathbf{H}],$$

transformed with the aid of the condition $\operatorname{div} \mathbf{H} = 0$, is written in the form

$$\frac{\partial H_r}{\partial t} + \frac{v}{r^2}\frac{\partial}{\partial r}(r^2 H_r) = 0, \qquad \frac{\partial H_\theta}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(rvH_\theta) = 0,$$

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where r is the radius; θ is the latitude, measured from the direction of the vector $\mathbf{H}_1$; H_r and H_θ are the radial and transverse components of the magnetic field; v is the velocity of motion of the gas, known from the solution of the self-similar problem of an explosion ([1]).

In the approximation under consideration, the conditions on the spherical shock wave for the components of the magnetic field have the form

$$H_{r2} = H_{r1} = H_1 \cos\theta, \qquad H_{\theta 2} = \frac{\gamma+1}{\gamma-1} H_{\theta 1} = \frac{\gamma+1}{\gamma-1} H_1 \sin\theta, \tag{2}$$

where the index 2 refers to quantities behind the shock front, and the index 1 to quantities ahead of the shock wave.

After substituting into system (1) the known ([1]) expression for v, the solution of this system is found by separation of variables. The required solution, satisfying conditions (2), can be written in the form

$$H_r = H_1 C_1(\gamma)(2-5\nu)^{\beta_1} \left(\frac{5\gamma}{2}\nu - 1\right)^{\beta_2} \left(1 - \frac{1-3\gamma}{2}\nu\right)^{\beta_3} \cos\theta,$$

(3)

$$H_\theta = H_1 C_2(\gamma)(2-5\nu)^{\beta_4} \left(\frac{5\gamma}{2}\nu - 1\right)^{\beta_2} \left(1 - \frac{1-3\gamma}{2}\nu\right)^{\beta_3} \sin\theta.$$

Here γ is the ratio of the specific heats; ν is a parameter varying within the limits $2/5\gamma \leqslant \nu \leqslant 4/5(\gamma+1)$ and related to r, t, the energy E_0, and the initial density ρ_1 by the relation

$$\frac{\alpha E_0 t^2}{\rho_1 r^5} = \frac{(\gamma+1)^2}{0.64} \nu^2 \left[\frac{5(\gamma+1)}{7-\gamma}\left(1 - \frac{3\gamma-1}{2}\nu\right)\right]^{\alpha_1} \left[\frac{\gamma+1}{\gamma-1}\left(\frac{5\gamma}{2}\nu - 1\right)\right]^{\alpha_2},$$

where α is a known constant ([1]),

$$\alpha_1 = \frac{13\gamma^2 - 7\gamma + 12}{(3\gamma-1)(2\gamma+1)}, \qquad \alpha_2 = \frac{5(1-\gamma)}{2\gamma+1},$$

$$\beta_1 = \frac{2\gamma}{3(\gamma-2)}, \qquad \beta_2 = \frac{2}{2\gamma+1}, \qquad \beta_3 = -\frac{2\alpha_1}{3(\gamma-2)}, \qquad \beta_4 = \frac{6-\gamma}{3(\gamma-2)},$$

$$C_1(\gamma) = \left[\frac{2(\gamma-1)}{\gamma+1}\right]^{-\beta_1} \left[\frac{\gamma-1}{\gamma+1}\right]^{-\beta_2} \left[\frac{3+11\gamma}{5(\gamma+1)}\right]^{-\beta_3},$$

$$C_2(\gamma) = \left[\frac{2(\gamma-1)}{\gamma+1}\right]^{-\beta_4} \left[\frac{\gamma-1}{\gamma+1}\right]^{-\beta_2} \left[\frac{3+11\gamma}{5(\gamma+1)}\right]^{-\beta_3} \left(\frac{\gamma+1}{\gamma-1}\right).$$

Taking into account that $\left(\frac{5\gamma}{2}\nu - 1\right)$ tends to zero as $\nu \to 2/5\gamma$ like r^{-5/α_2}, we find that $H_r \to 0$ and $H_\theta \to 0$ as $r \to 0$ like $r^{-5\beta_2/\alpha_2}$. For example, for $\gamma = 1.4$, $H_r \to 0$ as $r \to 0$ like r^5.

The currents arising in the region of gas motion are computed from the known formula

$$\mathbf{j} = \frac{c}{4\pi}\,\mathrm{rot}\,\mathbf{H}.$$

It turns out that $j_r = j_\theta = 0$, while the component j_φ is proportional to $\sin\theta$. The problem is solved analogously for the case of cylindrical and plane waves.

2. Strong shock waves propagating in a nonconducting medium in the presence of an electric and magnetic field emit electromagnetic waves ([3]). The presence of interaction between a strong shock wave and the electromagnetic field is associated with a sharp increase in the electrical conductivity of the gas behind the discontinuity front. For explosions of very great power, the increase in electrical conductivity may be due to thermal ionization of the gas. A significant increase in the electrical conductivity of the gas has also been observed for blast waves arising during the detonation of chemical explosives ([4]).

In the problems considered below it is assumed that behind the shock-wave front the conductivity of the gas is infinite, while ahead of it it is zero. In view of the weakness of the electromagnetic field, we shall not take into account the influence of the electric and magnetic fields on the motion of the gas.

As shown in works ([3,5]), the conditions at the shock-wave front with a discontinuity in conductivity, in the case where the electric and magnetic fields are parallel to the wave front, are the conditions of continuity of the magnetic and electric fields, if the coefficient of magnetic viscosity is greater than the coefficients of thermal conductivity and viscosity of the gas. The conditions for the gas-dynamic quantities remain the usual Hugoniot conditions.

Thus, for the intensities of the magnetic and electric fields we have $\mathbf{H}_1 = \mathbf{H}_2$, $\mathbf{E}_1 = \mathbf{E}_2$. In view of the infinite conductivity of the gas behind the discontinuity,

$$\mathbf{E}_2 = -\frac{1}{c}\left[(\mathbf{v}_2 - \mathbf{D})\mathbf{H}_2\right],$$

where $\mathbf{D}$ is the velocity of the shock wave. Consequently, at the shock-wave front the condition

$$\mathbf{E}_1 = -\frac{1}{c}\left[(\mathbf{v}_2 - \mathbf{D})\mathbf{H}_1\right]. \tag{4}$$

holds.

The processes associated with variation of the electric and magnetic fields in a medium with negligibly small conductivity are described by the equations

Maxwell' s equations, in which conduction currents are not taken into account:

$$\operatorname{rot}\mathbf{H} = \frac{1}{c}\frac{\partial E}{\partial t}, \qquad \operatorname{rot}\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{H}}{\partial t}, \qquad \operatorname{div}\mathbf{H} = 0, \qquad \operatorname{div}\mathbf{E} = 0. \tag{5}$$

Consider the case of plane waves. We shall use a Cartesian coordinate system, taking the axis oy to be directed parallel to the vector $\mathbf{E}$, and the axis oz parallel to the vector $\mathbf{H}$, with E and H in the undisturbed region assumed constant, equal to E_{y0}, H_{z0}; condition (4) takes the form

$$E_{y1} = -\frac{1}{c}(v_2 - D)H_{z1}. \tag{6}$$

From equations (5) we have

$$\frac{\partial E_y}{\partial x} = -\frac{1}{c}\frac{\partial H_z}{\partial t}, \qquad -\frac{\partial H_z}{\partial x} = \frac{1}{c}\frac{\partial E_y}{\partial t}. \tag{7}$$

It follows from (7) that E_y and H_z satisfy wave equations; moreover,

$$E_y + H_z = \Phi(\xi), \qquad E_y - H_z = F(\eta), \tag{8}$$

where $\xi = x - ct$, $\eta = x + ct$; $\Phi(\xi)$ and $F(\eta)$ are arbitrary functions; c is the speed of light.

Let us consider specific problems.

A. **Consideration of a plane shock wave.** Let, at the initial instant $t = 0$, a strong shock wave begin to propagate through the gas, with the pressure at its front varying according to the law

$$p_2 = p_0 \left(\frac{x_0}{x_2}\right)^{\beta}, \tag{9}$$

where p_0, β are positive constants; x_0 is the initial position of the shock wave; $x_2(t)$ is the coordinate of the shock wave. The case $\beta = 0$ in formula (9) was considered earlier in papers ([3],[5]). The gas-dynamic conditions on a strong wave have the form

$$v_2 = \frac{2}{\gamma + 1}D, \qquad D^2 = \frac{\gamma + 1}{2}\frac{p_2}{\rho_1} \quad \left(D = \frac{dx_2}{dt}\right). \tag{10}$$

From (9), (10) we obtain the law of motion of the shock wave

$$x_2 = \left[x_0 + (0.5\beta + 1) \left(\frac{p_0}{2} \frac{\gamma + 1}{\rho_1} \right)^{1/2} t \right]^{\frac{1}{1+0.5\beta}}. \quad (11)$$

The electromagnetic wave running ahead of the shock wave in the positive direction of the ox axis changes the initial field E_{y0}, H_{z0}, with

$$E_{1y} - H_{z1} = E_{y0} - H_{z0}. \quad (12)$$

At the shock-wave front E_{y1}, H_{z1} are related by relation (4), which, taking (10) into account, takes the form

$$E_{y1} = \frac{D}{c} \frac{\gamma - 1}{\gamma + 1} H_{z1}. \quad (13)$$

Using (8), (11), (12), (13), we find $H_{z1}(x, t), E_{y1}(x, t)$:

$$H_{z1} = \frac{E_{y0} - H_{z0}}{\left(\frac{p_0}{\rho_1} \frac{\gamma+1}{2} \right)^{1/2} \left(\frac{\gamma-1}{\gamma+1} \right) \left[\frac{x_0}{f(\xi)} \right]^{\beta/2} \frac{1}{c} - 1}, \qquad E_{y1} = H_{z1} + E_{y0} - H_{z0},$$

where $f(\xi)$ is found from the relation

$$\xi = f + c \left(\frac{2}{\gamma + 1} \right)^{1/2} \left[\left(\frac{f}{x_0} \right)^{1+0.5\beta} - 1 \right] \frac{1}{1 + 0.5\beta}.$$

Assuming the solution of the gas-dynamic problem to be completely known and using the frozen-in condition for the motion of the gas behind the shock-wave front,

$$H_z = \psi(S)\rho,$$

where $\psi(S)$ is an arbitrary function of the Lagrangian coordinate S, one can also find the dependence $H_z(x, t)$ or $H_z(S, t)$ in the region of gas flow. This dependence has the form

$$H_z = \frac{\gamma - 1}{\gamma + 1} \frac{E_{y0} - H_{z0}}{\rho_1} \frac{\rho(S, t)}{\left(\frac{\rho_0}{\rho_1} \frac{\gamma+1}{2} \right)^{1/2} \left(\frac{\gamma-1}{\gamma+1} \right) \left(\frac{S}{x_0} \right)^{\beta/2} \frac{1}{c} - 1}.$$

B. A strong explosion along a plane. At $t = 0$ an explosion of a charge in the form of a plane occurs. The solution of the gas-dynamic problem is known ([1]), and the law of motion of the shock wave is

$$x_2 = \left(\frac{E}{\rho_1}\right)^{1/3} t^{2/3},$$

where E is a constant related to the energy of the explosion.

We shall distinguish two cases in the solution of the problem.

1) $E_{y0} = 0$; $H_{z0} = 0$ for $0 \le x < x_0$; $H_{z0} \ne 0$ for $x > x_0$, where $x_0 \gg E/\rho_1 c^2$.
2) At the instant $t_0 > E/\rho_1 c^3$, a plane electromagnetic wave is incident on the shock wave arising in the explosion.

Case 1). Repeating arguments analogous to case A, we find: for $x < x_0$

$$H_{z1} = 0, \qquad E_{y1} = 0;$$

for $x \ge x_0$

$$H_{z1} = \frac{H_{z0}}{1 - \frac{2}{3c}\left(\frac{E}{\rho_1}\right)^{1/3} [\tau(\xi)]^{1/3} \frac{\gamma-1}{\gamma+1}}, \qquad E_{y1} = H_{z1} - H_{z0},$$

where $\tau(\xi)$ is determined from the equation

$$\tau = \frac{1}{c}\left[\left(\frac{E}{\rho_1}\right)^{1/3} \tau^{2/3} - \xi\right] \qquad (\tau \ge t_0).$$

Case 2). The solution of the problem of reflection of an electromagnetic wave from the surface of a shock wave can be found using the preceding derivations. It is only necessary to take into account that the quantities E_{y0} and H_{z0} will in this case not be arbitrary, but related by the relation

$$E_{y0} = H_{z0}.$$

This follows from the fact that before the collision of the shock and electromagnetic waves, the electric and magnetic fields in front of the shock-wave front were absent.

In conclusion, we note that the results obtained in Sec. 2 can be used to determine the parameters of a gas-dynamic shock wave if the parameters of the emitted electromagnetic wave are known.

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Note: Figure translations are in progress. See original paper for figures.