HYDROMECHANICS

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Abstract

Full Text

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THE CAUCHY–POISSON PROBLEM FOR WAVES OF FINITE AMPLITUDE

To the horizontal surface of an infinitely deep heavy liquid enclosed between two vertical walls there is imparted a certain initial velocity, varying from one point of the surface to another. It is required to determine the subsequent motion of the liquid, taking full account of the boundary conditions of the problem on the free surface of the liquid and not restricting ourselves, therefore, to the consideration of infinitesimal motions.

This problem can be solved with the aid of the Lagrangian variables a, b, and t, using the Lindstedt–Poincaré method from celestial mechanics.

Let $\xi(a, b, t)$, $\eta(a, b, t)$ denote the displacements of a liquid particle which at the initial instant has coordinates a, b, from its undisturbed position. To determine these displacements we have the equations:

$$\frac{\partial^2 \xi}{\partial t^2} = -\frac{\partial H}{\partial a} + \frac{D(\eta, H)}{D(a, b)},$$

$$\frac{\partial^2 \eta}{\partial t^2} = -\frac{\partial H}{\partial b} - \frac{D(\xi, H)}{D(a, b)},$$

$$\frac{\partial \xi}{\partial a} + \frac{\partial \eta}{\partial b} = -\frac{D(\xi, \eta)}{D(a, b)},$$

where $H = \frac{p}{\rho} + g(b + \eta)$; p is the hydrodynamic pressure of the liquid; ρ is the density; g is the acceleration of gravity.

This system of equations of the plane problem of hydrodynamics must be integrated under the following conditions:

$$\left.\begin{aligned} &\xi = \eta = 0, \\ &\frac{\partial \xi}{\partial t} = -\frac{\partial \varphi}{\partial a}, \\ &\frac{\partial \eta}{\partial t} = -\frac{\partial \varphi}{\partial b}, \end{aligned}\right\} \quad \text{for } t = 0,$$

$$p = p_0 = \text{const} \quad \text{for } b = 0, \qquad \xi = 0 \quad \text{on the walls of the basin.}$$

The function $\varphi = \varphi(a, b)$, the initial velocity potential, is a prescribed harmonic function of the variables a, b. The posed problem can be solved, with known simplicity, in the case when the function $\varphi(a, b)$ is defined by the series

$$\varphi(a, b) = \sum_{n=1}^{\infty} \varepsilon^n A_n e^{nb} \cos na,$$

in which the A_n are given numbers, and ε is a small parameter.

With such a choice of the initial velocity potential it is possible to compute the coefficients of the series

$$\begin{aligned} \xi &= \varepsilon\xi_1 + \varepsilon^2\xi_2 + \cdots, \\ \eta &= \varepsilon\eta_1 + \varepsilon^2\eta_2 + \cdots, \\ H &= \varepsilon H_1 + \varepsilon^2 H_2 + \cdots, \end{aligned} \tag{1}$$

satisfying the conditions and equations of the problem. These coefficients $\xi_1, \xi_2, \dots, \eta_1, \eta_2, \dots, H_1, H_2, \dots$ are functions of the Lagrange variables a, b, t.

We determine these coefficients in the form of trigonometric series containing no secular terms in time and arranged according to the arguments $w_1, \dots, w_2, \dots$, proportional to the time t.

We set

$$w_i = (\sigma_{i0} + \varepsilon\sigma_{i1} + \varepsilon^2\sigma_{i2} + \cdots)t \qquad (i = 1, 2, \dots).$$

The unknown quantities $\sigma_{i0}, \sigma_{i1}, \sigma_{i2}, \dots$ are determined from the imposed integration conditions.

In view of the fact that determining the functions $\xi_1, \xi_2, \dots, \eta_1, \eta_2, \dots, H_1, H_2, \dots$ and the numbers σ requires complicated calculations, in which errors can easily be made, the stated problem was solved at the same time by another method. We took the equations of hydrodynamics in Weber’ s form

$$\left(1 + \frac{\partial\xi}{\partial a}\right)\frac{\partial\xi}{\partial t} + \frac{\partial\eta}{\partial t}\frac{\partial\eta}{\partial a} = -\frac{\partial\varphi}{\partial a} - \frac{\partial\chi}{\partial a},$$

$$\frac{\partial\xi}{\partial t}\frac{\partial\xi}{\partial b} + \left(1 + \frac{\partial\eta}{\partial b}\right)\frac{\partial\eta}{\partial t} = -\frac{\partial\varphi}{\partial b} - \frac{\partial\chi}{\partial b},$$

where

$$\chi = \int_0^t \left\{ \frac{p - p_0}{\rho} + g(b + \eta) - \frac{1}{2} V^2 \right\} dt,$$

and V is the magnitude of the fluid velocity. With the aid of these equations we found the coefficients of the series written above (1).

Here the function $\varphi(a, b)$ was specified in the form of a sine series in the variable a. The integrals of the hydrodynamic equations obtained by the two different methods were compared, and their identity was established.

As a result of the computations performed, the coefficients of all series (1), accompanying powers of ε from the first through the fourth inclusive, were found.

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Note: Figure translations are in progress. See original paper for figures.