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Abstract

Full Text

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RADIATION OF FAST PARTICLES IN AN INHOMOGENEOUS MEDIUM

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PHYSICS

The article investigates radiation arising when a fast particle moves in an inhomogeneous medium. Let us first consider the case in which the inhomogeneities are arranged periodically. We shall characterize the properties of the medium by the dielectric constant ε, equal to

$$\varepsilon(\omega) = \varepsilon_0(\omega) + \Delta\varepsilon_1(\omega, z), \tag{1}$$

where ε_1 is a periodic function with period l, expandable in a Fourier series and satisfying the condition

$$\Delta\varepsilon_1 \ll \varepsilon_0. \tag{2}$$

Let the particle move uniformly and rectilinearly along the z-axis with velocity v (generalizing the calculations to the case of an arbitrarily directed velocity, as well as a velocity depending in an arbitrary way on the variable z, presents no difficulty). It is obvious that the presence of inhomogeneities leads to the appearance of radiation for exactly the same reason that leads to the scattering of light in an inhomogeneous medium. However, the presence of periodically arranged inhomogeneities must lead to interference phenomena, and the possibility of the appearance of appreciable radiation in a periodic medium is connected, as we shall now see, with the fulfillment of certain relations. To obtain them we shall follow the arguments given in ([1]), applying them to the case of uniform and rectilinear motion. We shall start from the law of conservation of energy

$$\delta E - \hbar\omega = 0 \tag{3}$$

and the law of conservation of momentum **along the motion of the particle**. The momentum transferred to the medium along the direction of motion must be equal to an integer multiple of $2\pi\hbar/l$—the "momentum of the periodic medium" :

$$\frac{v\delta p}{v} - \frac{\hbar\omega}{c}\sqrt{\varepsilon_0}\cos\theta = \frac{2\pi\hbar}{l}\,n. \tag{4}$$

In formulas (3) and (4), δE and δp denote the changes in the energy and momentum of the particle during radiation (we assume that $\hbar\omega \ll E_0$); θ is the angle between the emitted quantum and the initial motion. From relations (3) and (4) we obtain the basic condition under which radiation can arise in the inhomogeneous medium under consideration:

$$\omega_{\text{eff}} = \omega\left(1 - \frac{v}{c}\sqrt{\varepsilon_0}\cos\theta\right) = 2\pi n\frac{v}{l}. \tag{5}$$

Condition (5) has a simple visual meaning. For radiation to appear, it is necessary that the "effective frequency" of the radiation be in resonance with the frequency with which the particle traverses one period of the medium.

In the limit $l \to \infty$, the resonance condition (5) leads to the well-known Cherenkov–Vavilov condition. However, unlike the case $l \to \infty$, the condition (5) can also be satisfied in the region of frequencies where ε is less than unity and has the form

$$\varepsilon_0 = 1 - \frac{4\pi NZe^2}{m_e\omega^2}, \qquad \Delta = \frac{4\pi N'Z'e^2}{m_e\omega^2}, \qquad N'Z' \ll NZ. \tag{6}$$

To avoid misunderstandings, let us note that in deriving (5) we neglected the influence of $\Delta\varepsilon_1$ on the properties of the emitted quantum. The estimate (5) remains valid, in any case, if

$$\left|\frac{\Delta}{2\varepsilon_0^{1/2}}\frac{\omega}{c}\cos\theta\,\frac{l}{n}\,\frac{1}{2\pi}\right| < 1. \tag{7}$$

We shall see below (see conditions (9)) that condition (7) is fulfilled automatically. In addition, we assumed that the angle of emission of the quantum is θ and that it is constant over the length effective in emission (equal to $l_{\text{eff}} \simeq l$). It is easy, however, to verify that the "refraction" of the quantum may be neglected if (7) is fulfilled. Condition (5) and the requirement that $|\cos\theta| \leqslant 1$ immediately give us the interval of wavelengths emitted resonantly:

$$\frac{c}{v\sqrt{\varepsilon_0}}\left(1 + \frac{v}{c}\sqrt{\varepsilon_0}\right) \gg \frac{2\pi\lambda}{l}\,n \gg \frac{c}{v\sqrt{\varepsilon_0}}\left(1 - \frac{v}{c}\sqrt{\varepsilon_0}\right), \tag{8}$$

where $\lambda = c/\omega\sqrt{\varepsilon_0}$. The calculation which we shall give below shows that the harmonics with small n have the greatest intensity. It is seen from (8) that the case $n = 0$ can occur only if the right-hand side can vanish. This is the case of Cherenkov-Vavilov radiation in a periodic medium.

Let us consider the emission of frequencies that considerably exceed the characteristic frequencies of the medium. Then we may use the values of the dielectric constant in the form (6). For such frequencies $\varepsilon_0 \simeq 1$, and for $v \to c$ and $n = 1$ we can rewrite the right-hand side of inequality (8) in the form

$$\frac{E}{\hbar} \gg \frac{4\pi c}{l} \left(\frac{E}{mc^2}\right)^2 \gtrsim \omega \gtrsim \frac{4\pi N Z e^2}{m_e} \frac{l}{4\pi c}. \tag{9}$$

Let us note that the left-hand side of inequality (8) leads to the condition

$$\omega \gg \frac{\pi c}{l}, \tag{10}$$

which, for

$$l \gg 2\pi \left(\frac{m_e c^2}{4\pi N Z e^2}\right)^{1/2},$$

follows from (9).

From condition (9) there follows the existence of an energy threshold for the appearance of resonant radiation. The threshold is determined by the condition

$$\frac{E}{mc^2} \gtrsim \sqrt{\frac{4\pi N Z e^2}{m_e} \frac{l}{2\pi c}} \gg 1. \tag{11}$$

Let us turn to the calculation of the intensity of the radiation under consideration. For this purpose we shall use the macroscopic Maxwell equations for an inhomogeneous medium whose properties vary only in one direction.

For the component of the vector potential A_z ($A_x = A_y = 0$; this determines the polarization of the radiation)

$$A_z(x, y, z, t) = \int A(q, z, \omega) \exp(ik_{xx} + ik_{yy} - i\omega t)\, dk_{xdk} y d\omega, \tag{12}$$

where $q = \sqrt{k_x^2 + k_y^2}$, we have the equation (see, for example, ([3]))

$$\varepsilon \frac{\partial}{\partial z} \left(\frac{1}{\varepsilon} \frac{\partial A(q, z)}{\partial z}\right) + \left(\frac{\omega^2}{c^2}\varepsilon - q^2\right) A(q, z) = -\frac{e}{2\pi^2 c} \exp\left(i\frac{\omega}{v} z\right). \tag{13}$$

Equation (13), for an arbitrary dependence of ε on z, can be solved in general form in the geometrical-optics approximation (quasiclassical approximation). The solution for waves diverging from the source has the form

$$A(q,z,\omega)=\frac{iev}{4\pi^2 c}\left[\frac{\varepsilon(z)}{\chi(z)}\right]^{1/2}\int\frac{\exp\left(i\omega t+i\left|\int_{vt}^{z}\chi\,dz\right|\right)}{[\chi(vt)\varepsilon(vt)]^{1/2}}\,dt. \tag{14}$$

We have introduced the quantity

$$\chi=\sqrt{\frac{\omega^2}{c^2}\varepsilon_0-q^2+\frac{\omega^2}{c^2}\Delta\varepsilon_1}\simeq\chi_0+\frac{\chi_0\Delta\varepsilon_1}{2\varepsilon_0\cos^2\theta}, \tag{15}$$

where χ_0 is an expression analogous to χ for $\Delta\varepsilon_1=0$: $\chi_0=\frac{\omega}{c}\sqrt{\varepsilon_0}\cos\theta$, $q=\frac{\omega}{c}\sqrt{\varepsilon_0}\sin\theta$. We consider only real values of χ, corresponding to radiation. For values of χ close to zero (turning points), the solution loses its validity. The parameter of the quasiclassical expansion, if ε_1 is chosen in the form

$$\varepsilon_1=\cos\frac{2\pi z}{l}, \tag{16}$$

is

$$\left|\frac{\pi}{\cos^3\theta}\frac{\lambda}{l}\frac{\Delta}{\varepsilon_0}\right|\ll 1. \tag{17}$$

For us, however, fulfillment of the stronger condition will in fact be necessary,

$$\lambda\ll l \tag{17'}$$

($\cos\theta$, for the cases of interest to us, is always of order unity).

Let us consider the flux of the Poynting vector through a plane located at a distance $z\to\infty$ from the origin. Then it is easy to see that the radiation intensity in the interval $d\omega$ in the interval of angles $d\theta$ is given by the expression

$$dS=\frac{8\pi^3}{c^3}\varepsilon^{3/2}\omega^4\sin^3\theta\cos^2\theta\,|A(q,z)|^2\,d\theta\,d\omega. \tag{18}$$

The calculation of $A(q,Z)$ for the medium under consideration with dielectric constant given by formula (16) is very simple. To this end it is necessary to note that, under condition (17′), the dependence of the denominator on t may be neglected, and the integral in the exponent of expression (14) is readily evaluated if one uses expansion (15). The integral over t then leads to a single

δ-function (if (7) is satisfied with a large margin), i.e., to condition (5), derived from intuitive considerations (for $n = 1$). In calculating (18), one of the δ-functions must be replaced by $T/2\pi$, where T is the total flight time of the particle.

After integration over the angles of the emitted quantum, we obtain a formula for the number of photons emitted per unit path length under the condition $N'Z' \ll NZ$

$$dN = \frac{\pi N'Z' r_0^2}{137} \frac{d\omega}{\omega} \frac{N' l c^2}{\omega^2} \left[1 - \frac{l}{2\pi\lambda} \left(1 - \frac{v}{c} + \frac{2\pi N Z e^2}{m_e \omega^2} \right) \right]; \qquad (19)$$

the quantity r_0 is the classical radius of the electron. The formula for the number of emitted resonant quanta of particles with arbitrary mass is written in a form that permits easy comparison with the number of bremsstrahlung quanta emitted by an electron per unit path length. If $N'Z' \simeq NZ$, then the contribution of higher harmonics must be calculated in an analogous manner.

The total number of quanta is obtained by integrating expression (19) over the range of frequencies satisfying (8). Resonance radiation can be used for the detection of particles of ultrahigh energies. The general case of a periodic medium can be investigated in an analogous way. The case of a stratified medium (compare with (2)) with sharp boundaries leads to a formula that differs from formula (19) by the factor $16/\pi^2$ (for $N'Z' \ll NZ$ and equal thicknesses). It should be noted that the sharpness of the boundaries does not affect the result, and therefore one can always choose such a smooth transition that, on the one hand, the geometrical-optics approximation remains applicable, while, on the other hand, the presence of the smooth transition would not change the results. Let us also note that the effect of multiple scattering on the radiation under consideration can be neglected if one considers radiation of frequencies

$$\omega \ll \left(\frac{E}{E_s} \right)^2 \frac{cL}{l^2}, \qquad E_s \approx 21 \text{ MeV},$$

where L is the radiation length in centimeters. We do not give here the corresponding formulas for the case $N'Z' \approx NZ$ and for different layer thicknesses, for lack of space.

In conclusion, we note that radiation will also arise in the presence of random inhomogeneities. Investigation of the three-dimensional case leads to a formula close to formula (19), in which, however, l is the correlation length. Naturally, the angular distribution of the radiation will become directed as the energy of the emitted quantum increases.

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Note: Figure translations are in progress. See original paper for figures.