ON THE NUMERICAL SOLUTION OF THE EQUATION OF THE MAGNETIC FIELD IN IRON WITH ALLOWANCE FOR SATURATION

1960

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Fig. 1

Figure 1: Fig. 1

Abstract

Full Text

MATHEMATICAL PHYSICS

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ON THE NUMERICAL SOLUTION OF THE EQUATION OF THE MAGNETIC FIELD IN IRON WITH ALLOWANCE FOR SATURATION

(Presented by Academician M. V. Keldysh, January 3, 1960)

In the plane case, the equation for the magnetic potential $u(x, y)$ in iron has the form

$$\frac{\partial[\varphi\,\partial u/\partial x]}{\partial x} + \frac{\partial[\varphi\,\partial u/\partial y]}{\partial y} = 0, \tag{1}$$

where $\varphi = \varphi\,(|\operatorname{grad} u|^2)$ is the reciprocal of the magnetic permeability of iron, $\varphi = 1/\mu$. The boundary conditions are the usual ones; in what follows they play no role.

Fig. 1

Equation (1) is naturally reduced to the finite-difference form (see Fig. 1):

$$\left.\frac{\partial u}{\partial x}\right|_{\alpha} = u(0) - u(A), \qquad \left.\frac{\partial u}{\partial x}\right|_{\gamma} = u(C) - u(0),$$

$$\left.\frac{\partial u}{\partial y}\right|_{\beta} = u(B) - u(0), \qquad \left.\frac{\partial u}{\partial y}\right|_{\delta} = u(0) - u(D); \tag{2}$$

$$\varphi(\alpha) = \varphi\left\{[u(0) - u(A)]^2 + \left(\frac{1}{4}\right)^2 [u(B) + u(E) - u(G) - u(D)]^2\right\}, \tag{3}$$

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$$\varphi(\delta) = \varphi\left\{[u(D) - u(0)]^2 + \left(\frac{1}{4}\right)^2 [u(A) + u(G) - u(C) - u(H)]^2\right\}.$$

and, finally,

$$\left[\left(\varphi\frac{\partial u}{\partial x}\right)_\gamma - \left(\varphi\frac{\partial u}{\partial x}\right)_\alpha\right] + \left[\left(\varphi\frac{\partial u}{\partial y}\right)_\beta - \left(\varphi\frac{\partial u}{\partial y}\right)_\delta\right] = 0 \tag{4}$$

or, according to (2),

$$u_{\text{new}}(0) = K(\alpha)u(A) + K(\beta)u(B) + K(\gamma)u(C) + K(\delta)u(D), \tag{5}$$

where

$$K(\alpha) = \frac{\varphi(\alpha)}{\varphi(\alpha) + \varphi(\beta) + \varphi(\gamma) + \varphi(\delta)}, \dots, \quad K(\delta) = \frac{\varphi(\delta)}{\varphi(\alpha) + \varphi(\beta) + \varphi(\gamma) + \varphi(\delta)}. \tag{6}$$

Equations (5) and (6) are the equations of a stationary heat distribution with a variable coefficient of thermal conductivity φ, having the usual form shown in Fig. 2. However, for large gradients formulas (5) and (6) do not give a convergent process. The introduction of retardation by, for example, replacing equation (1) by

$$\frac{\partial u}{\partial t} = \frac{\partial[\varphi\,\partial u/\partial x]}{\partial x} + \frac{\partial[\varphi\,\partial u/\partial y]}{\partial y} \tag{1^A}$$

leads, of course, to the goal, but the time step has to be taken small, so that the present speeds of electronic machines prove insufficient.

Fig. 2

The following algorithm is proposed for the numerical solution of such problems.

For each point O, the values of the neighboring points $u(A), \dots, u(C)$ are fixed. One seeks $u_{\text{new}}(0)$ such that equation (5) is satisfied under the condition that in (3), instead of $u(0)$, there stands $u_{\text{new}}(0)$. (We shall call such modified conditions (3) by (3^A).) This procedure may be called a locally inverse step.

Now the process converges, in the sense of the number of iterations, faster than the analogous process for the Laplace equation, provided only that φ is a monotonically nondecreasing function of its argument. This is trivial if equation (1) is regarded as a heat equation, and the monotonicity of φ as an increase of thermal conductivity with the temperature gradient.

Fig. 3

For actual computations there is no need to solve equations (5), (6), and (3^A) simultaneously. For example, it is quite sufficient to put

$$u_1(0) = K(\alpha)u(A) + \dots,$$

where $K(\alpha), \dots, K(\delta)$ have been computed from (3);

$$u_2(0) = K_1(\alpha)u(A) + \dots,$$

where $K_1(\alpha), \dots, K_1(\delta)$ have been computed from (3) when $u(0)$ is replaced by $u_1(0)$, and, finally,

$$u_{\text{new}}(0) = u(0) + \frac{[u_2(0) - u_1(0)]^2}{2u_1(0) - u(0) - u_2(0)},$$

i.e., to make two iterations with subsequent interpolation:

$$u_1(0) = f[u(0)], \qquad u_2(0) = f[u_1(0)].$$

We seek $u_{\text{new}}(0)$ so that the straight line passing through $u(0)$, $u_1(0)$ and $u_1(0)$, $u_2(0)$ intersects, at $u_{\text{new}}(0)$, $u_{\text{new}}(0)$, the bisector of the first quadrant (Fig. 3).

The corresponding calculations were carried out on Bessonov' s relay computing machine (RCM).

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Received
3 XI 1959

Note: Figure translations are in progress. See original paper for figures.