



Soviet-era science, translated into English

MECHANICS

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1960

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Abstract

Full Text

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ON A CLASS OF COUPLER CURVES

The well-known ⁽¹⁾ coupler curves may be regarded as curves enveloping a series of positions of individual points belonging to the coupler plane. Below we consider coupler curves that are envelopes of straight lines belonging to the coupler plane. Let an arbitrary straight line $u - u$ be chosen in the plane of the coupler BC (Fig. 1). From the point A drop the perpendiculars AF and AE to the lines $u - u$ and BC . Then the equation of the family of lines $u - u$ will be

$$x \cos \theta + y \sin \theta = p \quad (1)$$

or

$$x \cos(\theta_1 + \gamma) + y \sin(\theta_1 + \gamma) = p_1 \cos \gamma + \left(\sqrt{a^2 - p_1^2} - e \right) \sin \gamma, \quad (2)$$

where $p = AF$, $p_1 = AE$, $e = BG$, and $a = AB$.

Let us find the partial derivative with respect to the parameter θ_1 of expression (2)

$$y \cos(\theta_1 + \gamma) - x \sin(\theta_1 + \gamma) = \frac{\partial p_1}{\partial \theta_1} \cos \gamma - \frac{p_1}{\sqrt{a^2 - p_1^2}} \frac{\partial p_1}{\partial \theta_1} \sin \gamma. \quad (3)$$

The segment p_1 can be expressed in terms of the parameters of the mechanism (Fig. 1). Introduce the notation: $AB = a$, $BC = b$, $CD = c$, $DA = d$, $a^2 + b^2 + d^2 - c^2 = m^2$, $2bd = k^2$, $2ad = r^2$, $2ab = t^2$, and $d^2 + b^2 = n^2$. Then the segment p_1 can be represented as follows:

$$p_1 = \frac{d(m^2 - k^2 \sin \theta_1) \cos \theta_1}{2(n^2 - k^2 \sin \theta_1)} \pm \quad (4)$$

$$\pm \sqrt{\frac{d^2(m^2 - k^2 \sin \theta_1)^2 \cos^2 \theta_1 - (n^2 - k^2 \sin \theta_1) [(m^2 - k^2 \sin \theta_1)^2 - 4a^2(b - d \sin \theta_1)^2]}{4(n^2 - k^2 \sin \theta_1)^2}}.$$

The sign of the second term of expression (4) is chosen depending on the assembly conditions of the mechanism, since to each position of the link AB there correspond two possible positions of the remaining links. In what follows, the position shown in Fig. 1 is considered.

The partial derivative $\partial p_1/\partial\theta_1 = \Psi$ will be equal to

$$\Psi = R + T + P + Q + S, \quad (5)$$

where

$$R = \frac{d \left[k^2 (\sin^2 \theta_1 - \cos^2 \theta_1) - m^2 \sin \theta_1 \right]}{2(n^2 - k^2 \sin \theta_1)} + \frac{dk^2 \cos \theta_1 (m^2 - k^2 \sin \theta_1)}{(n^2 - k^2 \sin \theta_1)^2},$$

$$T = \frac{bd \cos \theta_1 (m^2 - k^2 \sin \theta_1) [2n^2 + (m^2 - k^2 \sin \theta_1)]}{2(n^2 - k^2 \sin \theta_1) \sqrt{d^2 \cos^2 \theta_1 (m^2 - k^2 \sin \theta_1)^2 - (n^2 - k^2 \sin \theta_1) [(m^2 - k^2 \sin \theta_1) - 4a^2 (b - d \sin \theta_1)^2]}},$$

$$P = \frac{d^2 (m^2 - k^2 \sin \theta_1) \cos \theta_1 [(m^2 - k^2 \sin \theta_1) + bd \cos^2 \theta_1]}{2(n^2 - k^2 \sin \theta_1) \sqrt{d^2 \cos^2 \theta_1 (m^2 - k^2 \sin \theta_1)^2 - (n^2 - k^2 \sin \theta_1) [(m^2 - k^2 \sin \theta_1) - 4a^2 (b - d \sin \theta_1)^2]}},$$

$$Q = \frac{2a^2 d \cos \theta_1 (b - d \sin \theta_1) [b(b - d \sin \theta_1) + (n^2 - k^2 \sin \theta_1)]}{(n^2 - k^2 \sin \theta_1) \sqrt{d^2 \cos^2 \theta_1 (m^2 - k^2 \sin \theta_1)^2 - (n^2 - k^2 \sin \theta_1) [(m^2 - k^2 \sin \theta_1) - 4a^2 (b - d \sin \theta_1)^2]}},$$

$$S = \frac{bd \cos \theta_1 \sqrt{d^2 \cos^2 \theta_1 (m^2 - k^2 \sin \theta_1)^2 - (n^2 - k^2 \sin \theta_1) [(m^2 - k^2 \sin \theta_1) - 4a^2 (b - d \sin \theta_1)^2]}}{(n^2 - k^2 \sin \theta_1)^2}.$$

From equations (2) and (3), taking into account equation (5), we obtain the parametric equations for the envelope curve in the form $x = x(\theta_1)$ and $y = y(\theta_1)$. We have

$$x = [p_1 \cos(\theta_1 + \gamma) - \Psi \sin(\theta_1 + \gamma)] \cos \gamma + \left[\left(\sqrt{a^2 - p_1^2} - e \right) \cos(\theta_1 + \gamma) + \frac{p_1 \Psi}{\sqrt{a^2 - p_1^2}} \sin(\theta_1 + \gamma) \right] \sin \gamma; \quad (6)$$

$$y = [p_1 \sin(\theta_1 + \gamma) + \Psi \cos(\theta_1 + \gamma)] \cos \gamma +$$

Fig. 1

Figure 1: Fig. 1

Fig. 2

Figure 2: Fig. 2

$$+ \left[\left(\sqrt{a^2 - p_1^2} - e \right) \sin(\theta_1 + \gamma) - \frac{p_1 \Psi}{\sqrt{a^2 - p_1^2}} \cos(\theta_1 + \gamma) \right] \sin \gamma. \quad (7)$$

It follows from equations (6) and (7) that any straight line of the connecting-rod plane parallel to the line $u - u$ will have, as its envelope, the curve defined by equations (6) and (7), and thus, for the study of envelopes, one may set $e = 0$ in equations (6) and (7), i.e., consider the case when the line $u - u$ passes through point B . If the direction of the line $u - u$ coincides with the direction BC of the connecting rod, then the angle $\gamma = 0$, and equations (6) and (7) take the form

Fig. 1

Fig. 2

$$x = p_1 \cos \theta_1 - \Psi \sin \theta_1, \quad (8)$$

$$y = p_1 \sin \theta_1 + \Psi \cos \theta_1, \quad (9)$$

where p_1 and Ψ are determined from equations (4) and (5).

If the link AB is chosen as the driving link, then the angle of rotation α of this link (Fig. 1) is related to the angle θ_1 by the condition

$$\cos \theta_1 = \frac{m^2 - k^2(r^2 - t^2) \cos \alpha}{t^2 \sin \alpha}. \quad (10)$$

The class of curves considered may be called **linearly enveloping connecting-rod curves**, in which case the chosen line is a straight line. The construction of these curves by points may be carried out as follows. We find (see Fig. 2) the position of the instantaneous center of rotation of link P , and from point P drop a perpendicular to the line $u - u$. Point G will be a point belonging to the curve, the connecting-rod envelope of the straight line $u - u$. The functions $p_1 = p_1(\theta_1)$ and $\Psi = \Psi(\theta_1)$, defined by equations (4) and (5), can be represented geometrically. The segment $AE = p$ (Fig. 1) is made in the form of the link AN , sliding in the slider M , whose guides form a right angle. We then construct a rotated plane of velocities $p^l c$ of the basic mechanism $ABCD$, and then find the relative

sliding velocity of the link M along the axes of the links BC and AN . The segment pt , rotated through an angle of 90° , is proportional to the quantity Ψ in the considered position of the mechanism.

The loci of the points F and E (Fig. 1) will be the polodes of the considered line envelopes of connecting-rod curves, if point A is chosen as the pole of the polode. The polode that is the locus of the points E can be reproduced mechanically as the trajectory of the point E of the slider M (Fig. 2). The equations of this polode in parametric form are

$$x = -p_1 \cos \theta_1, \quad (11)$$

$$y = p_1 \sin \theta_1, \quad (12)$$

where p_1 is determined by equation (4). The equations of the polode described by the point F (Fig. 2) are:

$$x = -\left(p_1 \cos \gamma + \sqrt{a^2 - p_1^2 \sin^2 \gamma}\right) \cos(\theta_1 + \gamma), \quad (13)$$

$$y = \left(p_1 \cos \gamma + \sqrt{a^2 - p_1^2 \sin^2 \gamma}\right) \sin(\theta_1 + \gamma), \quad (14)$$

where p_1 is likewise determined by equation (4).

Received
21 I 1959

CITED LITERATURE

1. I. I. Artobolevskii, N. I. Levitskii, S. A. Cherkudinov, *Synthesis of Plane Mechanisms*, 1959.

Note: Figure translations are in progress. See original paper for figures.

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