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PHYSICS

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ON THE PHENOMENON OF MAGNETIC PUSHING-ASIDE IN A FREE-MOLECULAR PLASMA FLOW

(TOWARD THE THEORY OF THE FLOW OF SOLAR CORPUSCULAR STREAMS AROUND THE EARTH' S MAGNETIC DIPOLE)

(Presented by Academician L. I. Sedov on 9 VII 1960)

In the present work we analyze the case of the flow, by a free-molecular plasma stream, around bodies possessing their own magnetic field, when the principal linear parameter of the interaction is large in comparison with the thickness of the "returning" layer and greater than the dimensions of the body (i.e., the body is entirely located in the cavity S). The general problem of magnetic "pushing-aside" in this case is formulated. The results of the consideration are applied to the problem of the flow of solar corpuscular streams around the Earth' s magnetic dipole in the case when they are free-molecular.

§ 1. Suppose there is a free-molecular stream T_1 (i.e., a stream in which particle collisions are absent) of fully ionized plasma, incident with velocity $\mathbf{u}_0$ on a homogeneous magnetic field (see Fig. 1; the magnetic field $\mathbf{H}$ has one component along Oy, while the velocity $\mathbf{u}_0$ has an arbitrary orientation in space).

For certain values of the quantities $\mathbf{u}_0$, $\mathbf{H}$, and the stream density ρ_0, the following steady state may be established:

1) The magnetic field $\mathbf{H}$ differs from zero only in the region $x > 0$.

2) The motion of the medium is such that the particles, entering the zone $x > 0$, as a consequence of the presence of the magnetic field return to the region $x < 0$ and move away from the zone occupied by the magnetic field, so that the flow consists of the direct stream T_1 and the reverse stream T_2, which penetrate one another and do not interact with one another.

3) Owing to the different character of the motions of electrons and ions (because of the difference in the masses and charges of these particles), in the zone $x > 0$ there arises a self-consistent electric field $E(x)$, directed along the x-axis, and an electric current $j(x)$, flowing along the z-axis; the magnetic field is likewise self-consistent and varies from zero at $x = 0$ to H_0 at $x = l$, where l is of the order of less than the ion Larmor radius calculated for the field H_0. We shall call the layer $0 < x < l$ the **returning** layer. It is obvious that outside the returning layer the electric field is absent, and $H = H_0$ for $x > l$.

4) Owing to the symmetry of the Lorentz forces acting on the particles in the returning layer, the distribution functions f of the streams T_1 and T_2 are, obviously, related by the relation:

$$f_{T_1}(u, v, w) = f_{T_2}(-u, v, w). \tag{1}$$

In other words, the stream T_2 represents the mirror reflection, from the returning layer, of the stream T_1.

5) Applying the momentum theorem, we find the relation

$$2\rho_0 u_0^2 \cos^2(\widehat{\mathbf{u}_0, x}) = \frac{H_0^2}{8\pi}, \tag{2}$$

to which the quantities ρ_0, u_0, H_0 must satisfy on both sides of the return layer.

The theory of the return layer was first considered by Chapman and Ferraro (see the book [1]).

§ 2. Let us further suppose that there is a body possessing its own magnetic field, and let a free-molecular flow of fully ionized plasma impinge on this magnetic field. We shall also suppose that the thickness of the return layer is small in comparison with the characteristic linear dimension L of the interaction between the flow and the magnetic field ($l/L \ll 1$).

Then the flow past the magnetic field will be characterized by the following features:

1) The free-molecular plasma flow flows around a certain cavity S (see Fig. 2), inside which the magnetic field is enclosed; outside S there is no magnetic field.

2) The flow around the cavity S consists in the fact that the plasma particles are reflected elastically from the surface S, i.e., the component of the particle momentum normal to the surface S changes sign, while the tangential component remains unchanged.

Fig. 1

Fig. 1

Figure 1: Fig. 1

Fig. 2

Figure 2: Fig. 2

3) The boundary S is, strictly speaking, a return layer, the interaction within which is the same as in § 1, by virtue of the assumption made above that $l/L \ll 1$. By virtue of this same assumption, S is regarded as a surface. The return layer screens the magnetic field.

The phenomenon considered is analogous to the phenomenon of magnetic squeezing in classical magnetohydrodynamics and therefore may also be called the phenomenon of magnetic squeezing of a free-molecular plasma flow.

Fig. 2

If the plasma flow is not fully ionized, but is free-molecular, then the external magnetic field evidently squeezes out only the charged particles, and a third homogeneous flow T_3 is formed, freely penetrating into the cavity S and flowing around the body in the usual manner.

The mathematical formulation of the problem of finding the boundary S and the magnetic field inside S is as follows:

It is required to find the boundary S and the magnetic field $\mathbf{H}$, if:

A. On the boundary S

$$H^2/8\pi = 2\rho_0 u_0^2 \cos^2(\widehat{\mathbf{u}_0, \mathbf{n}}) \quad \text{for } \cos(\widehat{\mathbf{u}_0, \mathbf{n}}) \leq 0,$$

$$H = 0 \quad \text{for } \cos(\widehat{\mathbf{u}_0, \mathbf{n}}) > 0.$$

B. Inside S the field obeys the equations

$$\operatorname{div} \mathbf{H} = 0; \qquad \operatorname{rot} \mathbf{H} = 0.$$

C. The component of the magnetic field normal to the boundary S on S is equal to zero.

D. The specified singularities of the magnetic field are singular points inside the cavity S.

This problem coincides in a remarkable way with the problem of magnetic squeezing in classical magnetohydrodynamics in the case where the gas-dynamic pressure on the contour S can be found by Newton' s formula (see ([2,3])). The difference consists only in the fact that instead of the quantity p_0', close to ρu_0^2, here there appears the quantity $2\rho u_0^2$ (A), i.e., approximately twice as large. Therefore the corresponding results of ([2,3]) can be transferred to the case under consideration by replacing p_0' in the formulas by $2\rho_0 u_0^2$.

§ 3. Let us now consider the question of the flow around the Earth' s magnetic dipole by corpuscular streams of the Sun, if the latter have a density corresponding approximately to 10^2 particles in 1 cm^3, and a static temperature greater than 10^3 °K. In this case the stream may be regarded as free-molecular and monovelocity ($u_0 \sim 10^8$ cm/sec). The value $H_0 \sim 10^{-3} \div 10^{-2}$, which corresponds to ion Larmor radii of the order of 10^6 cm.

The characteristic length for the interaction L (the size of the cavity S) is of the order of 10^9 cm; thus the quantity $l/L < 10^{-3}$. Consequently, the interaction of the corpuscular stream with the Earth' s magnetic field has the character of the squeezing considered in § 2.

Applying the results of ([3]) to the case under consideration, we have:

1) The cavity S is a half-body oriented in the direction of the oncoming stream. The characteristic size

$$L \sim \left(\frac{M}{c}\right)^{1/3} \rho_0^{-1/6} u_0^{-1/3}.$$

(M is the magnitude of the Earth' s magnetic dipole). A force P_1 and a moment P_2 are applied to the magnetic dipole by the gas flowing around the cavity S, having the following orders of magnitude:

$$P_1 \sim \left(\frac{M}{c}\right)^{2/3} \rho_0^{1/3} u_0^{4/3}, \qquad P_2 \sim \frac{M}{c} \rho_0^{1/2} u_0^3.$$

2) In the regions where the Earth' s magnetic axis intersects the boundary of the cavity S, there are critical points, i.e., points where the magnetic-field intensity vanishes. The critical points, by virtue of condition A, are points of inflection of the surface S.

It should be noted that the behavior of particles in the neighborhood of the critical points requires additional consideration, since in the neighborhood of the critical points the basic requirement of the theory concerning the action of a plane returning layer on the particles is violated, and the mechanism of interaction will be different from that considered in § 1.

The possibility of the existence of a mechanism of particle capture inside the cavity S is not excluded; however, since a small mass of gas participates in it in comparison with the interacting mass, this allows one to hope for the correctness of the general character of the interaction considered.

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Note: Figure translations are in progress. See original paper for figures.