Corresponding Member of the Academy of Sciences of the USSR E. N. BLINOVA

1960

SovietRxiv

Abstract

Full Text

GEOPHYSICS

Corresponding Member of the Academy of Sciences of the USSR E. N. BLINOVA

HYDRODYNAMIC FORECAST OF MEAN MONTHLY TEMPERATURE ANOMALIES FOR THE NORTHERN HEMISPHERE OF THE EARTH USING DATA FROM THE INTERNATIONAL GEOPHYSICAL YEAR

In 1950 the author developed a method for long-range forecasting of temperature anomalies on the basis of solving the system of equations of hydrodynamics and thermodynamics. This method was tested under operational conditions up to the end of 1957 (see, for example, ([3])). In these tests the initial data entering into the forecast system were taken over a limited territory, the method of representing the initial fields of meteorological elements was imprecise, and the forecasts themselves were compiled and evaluated only for part of the territory of the northern hemisphere of the Earth. The International Geophysical Year made it possible to broaden the formulation of the problem. For the first time it became possible to use, under operational conditions, aerological observations sufficiently complete from the point of view of the planetary scale. Thus, in particular, the possibility became clear of a hydrodynamic forecast of temperature anomalies for the entire northern hemisphere. The theory and some results of its application in the light of the use of data from the International Geophysical Year are set forth in the present article.

§ 1. It is assumed that the motions of the atmosphere are close to purely zonal rotation. The latter is characterized by the circulation index α (the angular velocity of the motion of the air relative to the Earth) and by the temperature difference between the pole and the equator ([1,2]). The calculation is carried out for one hemisphere of the Earth; in doing so, the impermeability of air masses through the equator is approximately assumed (the blurring of the fields of meteorological elements at the equator). We represent the air temperature T in the form $T = \overline{T} + T' + T''$. Here $\overline{T}$ is the zonal temperature (obtained by averaging along a circle of latitude); T' is the mean climatic nonzonal part (quasi-stationary for the forecast period); T'' is the nonzonal nonstationary part —the "anomaly" of temperature. Our problem consists in finding T''. For this purpose the heat-inflow equation will serve us.

We take the Earth to be a sphere of radius a_0, and describe atmospheric motions in a system of spherical coordinates θ (complement of latitude), λ (longitude of the place), and z (height above sea level). The components of wind velocity along the axes θ and λ will be denoted respectively by v_θ and v_λ. We linearize the heat-inflow equation with respect to purely zonal circulation. This means that the convective terms of this equation

$$\frac{v_\lambda}{a_0 \sin\theta} \frac{\partial T}{\partial \lambda} + \frac{v_\theta}{a_0} \frac{\partial T}{\partial \theta} \tag{1}$$

we represent in the form

$$\frac{\overline{v}_\lambda}{a_0 \sin\theta} \frac{\partial T''}{\partial \lambda} + \frac{v''_\theta}{a_0} \frac{\partial \overline{T}}{\partial \theta}, \tag{2}$$

where $\overline{v}_\lambda$ is the velocity v_λ averaged along a circle of latitude; v''_θ is the nonstationary nonzonal part of the meridional velocity. We further assume

$$\overline{v}_\lambda = \alpha a_0 \sin\theta, \qquad \overline{T} = T_0 + M \sin^2\theta. \tag{3}$$

(α and M are constants). Then the heat inflow equation can approximately be written in the form

$$\frac{\partial \tau}{\partial t} + \alpha \frac{\partial \tau}{\partial \lambda} + 2M \frac{\sin\theta}{a_0} v''_\theta = \frac{k''}{a_0^2} \Delta\tau + \frac{\partial}{\partial z}\left(k' \frac{\partial \tau}{\partial z}\right). \tag{4}$$

Here $\tau = \dfrac{T''}{\sin\theta}$; t is time; $\Delta = \dfrac{1}{\sin\theta}\dfrac{\partial}{\partial\theta}\left(\sin\theta \dfrac{\partial}{\partial\theta}\right) + \dfrac{1}{\sin^2\theta}\dfrac{\partial^2}{\partial\lambda^2}$; k' and k'' are the coefficients of turbulent thermal conductivity in the vertical and horizontal directions, respectively.

Equation (4) takes into account the intrusion of heat and cold along the meridians, advection by the basic west-east currents, smoothing of temperature anomalies due to horizontal mixing, and transformation in the vertical.

Let us take for v''_θ in (4) the values of the meridional velocity at the mean level of the atmosphere, where a stream function can be introduced such that

$$v_\theta = -\frac{1}{a_0 \sin\theta} \frac{\partial \psi}{\partial \lambda}, \qquad v_\lambda = \frac{1}{a_0} \frac{\partial \psi}{\partial \theta}.$$

In this case

$$v''_\theta = -\frac{1}{a_0 \sin\theta} \frac{\partial \psi''}{\partial \lambda}$$

(ψ'' is the nonstationary nonzonal part of ψ).

The function ψ satisfies the vorticity-transfer equation

$$\frac{\partial \Delta\psi}{\partial t} + \frac{1}{a_0^2 \sin\theta}\left(\frac{\partial\psi}{\partial\theta}\frac{\partial\Delta\psi}{\partial\lambda} - \frac{\partial\psi}{\partial\lambda}\frac{\partial\Delta\psi}{\partial\theta}\right) + 2\omega\frac{\partial\psi}{\partial\lambda} = 0.$$

Hence, after linearization with respect to the flow (3), we obtain for ψ''

$$\frac{\partial\Delta\psi''}{\partial t} + \alpha\frac{\partial\Delta\psi''}{\partial\lambda} + 2(\alpha+\omega)\frac{\partial\psi''}{\partial\lambda} = 0. \tag{5}$$

This equation is solved independently of (4). The solution for ψ'' in the case of constant α can be represented in the form of the series (1)

$$\psi'' = \sum_{n=1}^{\infty}\sum_{m=1}^{n}\left[D_n^m \cos(m\lambda + \sigma_n^m t) + D_n'^m \sin(m\lambda + \sigma_n^m t)\right] P_n^m(\cos\theta), \tag{6}$$

where

$$\sigma_n^m = \frac{2(\alpha+\omega)m}{n(n+1)} - \alpha m \qquad (n-m \text{ odd}). \tag{7}$$

It follows from formula (6) that D_n^m and $D_n'^m$ are the coefficients of the expansion in spherical functions of the initial field ψ''. To determine D_n^m and $D_n'^m$, one can use the equation relating the values of the heights H of the 600-mb isobaric surface and the stream function:

$$2\left[\omega\cos\theta + \frac{1}{a_0}\frac{\partial v_\lambda}{\partial\theta}\right]\Delta\psi - 2\omega\sin\theta\frac{\partial\psi}{\partial\theta} - 2\left(\frac{\partial v_\theta}{\partial\theta}\right)^2 - 2\left(\frac{\partial v_\lambda}{\partial\theta}\right)^2 - v_\lambda^2 - v_\theta^2 = g\Delta H. \tag{8}$$

Linearizing equation (8), we obtain

$$\cos\theta\Delta\psi'' - \sin\theta\frac{\partial\psi''}{\partial\theta} = \frac{g}{2(\alpha+\omega)}\Delta H'', \tag{9}$$

where $H''(\theta,\lambda,t)$ is the nonstationary nonzonal part of H.

Equation (9) is valid for any instant of time. If $(H'')_{t=0}$ is represented in the form of the series

$$H''(\theta,\lambda,0) = \sum_{n=1}^{\infty}\sum_{m=1}^{n}\left(A_n^m \cos m\lambda + A_n'^m \sin m\lambda\right) P_n^m(\cos\theta) \tag{10}$$

Figure 3

Figure 1: Figure 3

Figure 4

Figure 2: Figure 4

Fig. 3. Predicted mean monthly anomalies of nonzonal temperatures for January 1959 ($M = 78°\mathrm{C},\ \alpha/\omega = 0.030 + 0.002$–mean value with correction for the annual trend)

Fig. 4. Mean monthly temperature anomalies. January 1959.

and take from (6) $(\psi'')_{t=0}$, then from (9) we obtain (see (2))

$$D_{n+1}^{m}\frac{(n+2)(n+m+1)}{(n+1)(2n+3)} + D_{n-1}^{m}\frac{(n-m)(n-1)}{(2n-1)n} = \frac{g}{2(\alpha+\omega)}A_{n}^{m}. \qquad (11)$$

The recurrence formula (11) makes it possible successively to determine D_n^m from A_n^m (similarly, $D_n'^m$ is determined from $A_n'^m$).

Thus, v_θ', entering into (4), may be regarded as a known function of θ, λ, and t, provided only that the initial field H is known for the middle level of the atmosphere.

§ 2. The problem reduces to determining τ from (4) for a known v_θ'. As boundary conditions with respect to z we take (apart from boundedness as $z \to \infty$) the condition of heat balance at the Earth,

$$-\lambda'\frac{\partial\tau}{\partial z} + \lambda^*\frac{\partial\tau^*}{\partial z} = S'' - \mu\tau$$

$$\text{at } z = 0, \qquad (12)$$

Fig. 1. Predicted mean monthly anomalies of nonseasonal temperatures for July 1958.
($M = 31°\mathrm{C},\ \alpha/\omega = 0.030$ –mean value)

where λ' is the coefficient of turbulent heat conductivity of air in the vertical; λ^* and $\cos\theta\,\tau^*$ are, respectively, the coefficient of turbulent heat conductivity and the temperature anomaly of the underlying surface; S'' is a quantity associated with the departure from normal of the heat input from the Sun; μ is a parameter

Figure 1

Figure 3: Figure 1

Figure 2

Figure 4: Figure 2

characterizing radiation of the underlying surface. The heat flux for evaporation is taken into account indirectly (through a change in the coefficient λ').

For closing the problem we shall also write the equation

$$\frac{\partial \tau^*}{\partial t} = k^* \frac{\partial^2 \tau^*}{\partial z^2} \tag{13}$$

Fig. 2. Mean monthly temperature anomalies.
July 1958.

(k^* is the coefficient of thermal diffusivity of the underlying surface). We shall seek the solution of equation (13) under the conditions:

$$\tau^* = \tau \quad \text{at } z = 0, \qquad \tau^* \text{ is bounded as } z \to -\infty. \tag{14}$$

At first glance it may seem that the initial field τ is of substantial importance for the forecast. But, because of turbulent dissipation, the influence of the initial τ already after several days ceases to make itself felt (decays). On the contrary, all new intrusions of heat or cold along the meridian (described by the term containing v'_θ in (4)) will occur throughout–

···over the entire forecast period. We shall seek the "steady regime," assuming that the initial values of τ have already decayed.*

The corresponding solution has the form (we take $\lambda', \lambda^*, k', k^*, k'', \mu$ to be constants)

$$\begin{aligned}
\tau &= \sum_{n=1}^{\infty} \sum_{m=1}^{n} \left[\tau_{1n}^m(z) \cos(m\lambda + \sigma_n^m t) + \tau_{2n}^m(z) \sin(m\lambda + \sigma_n^m t)\right] P_n^m(\cos\theta), \\
\tau^* &= \sum_{n=1}^{\infty} \sum_{m=1}^{n} \left[\tau_{1n}^{*m}(z) \cos(m\lambda + \sigma_n^m t) + \tau_{2n}^{*m}(z) \sin(m\lambda + \sigma_n^m t)\right] P_n^m(\cos\theta),
\end{aligned} \tag{15}$$

where, according to (4), (6), (12), (13), and (14),

$$\tau_{1n}^m + i\tau_{2n}^m = \frac{2M}{a_0^2} \left\{ 1 - \frac{a_n^m}{a_n^m + \sqrt{\tilde{\sigma}_n^m}} \exp\left[-(1-i)\sqrt{\frac{\tilde{\sigma}_n^m}{2k'}}\, z \right] \right\} \frac{m}{\tilde{\sigma}_n^m} \left(D_n^m + i D_n'^m \right), \tag{16}$$

$$\tau_{1n}^{*m} + i\tau_{2n}^{*m} = -\frac{2Mm}{a_0^2\sqrt{\tilde{\sigma}_n^m}}\,\frac{D_n^m + iD_n'^m}{a_n^m + \sqrt{\tilde{\sigma}_n^m}}\,\exp\left[(1-i)\sqrt{\frac{\tilde{\sigma}_n^m}{2k^*}}\,z\right], \qquad (17)$$

where

$$\tilde{\sigma}_n^m = \sigma_n^m + \alpha m + \frac{k'' n(n+1)}{a_0^2}\,i, \qquad a_n^m = \frac{\lambda^*}{\lambda'}\sqrt{\frac{k'}{k^*}\,\sigma_n^m} + \mu\frac{\sqrt{k'}}{\lambda'}\sqrt{i}.$$

§ 3. Formulas (15) are suitable for forecasting temperature anomalies at any height z and for any time t. We tested them in forecasts of mean monthly temperature anomalies at sea level for the entire Northern Hemisphere of the Earth. These forecasts have been issued under operational conditions, beginning in May 1958, at the Institute of Applied Geophysics of the Academy of Sciences of the USSR. The initial data are in each case 40 days removed from the beginning of the month for which the anomalies are being predicted. The coefficients D_n^m and $D_n'^m$ are determined by formulas (11) with the aid of $A_n^m, A_n'^m$; the latter are found from the expansion into series (10) of the initial values of the 600 mb surface heights for the Northern Hemisphere ($n \leqslant 36,\ m \leqslant 18$).

The parameters α and M must be chosen with particular care. We constructed all forecasts taking into account only the annual course of α and M, adopting, for each forecast, specially smoothed mean 70-day values of these parameters. The remaining parameters of the problem $(k', k'', \lambda', \lambda^*, k^*, \mu)$ were taken to be the same for all months of the year.

Examples of forecasts are given in Fig. 1 (forecast for July 1958) and in Fig. 3 (forecast for January 1959). Figures 2 and 4 give the corresponding observed temperature anomalies [4]. Isolines of negative anomalies are indicated by dashed lines in Figs. 1 and 2 and by blue lines in Figs. 3 and 4.

Is the meridional transport of heat adequately represented by means of only a single parameter M in our linear model? An indirect positive answer to this question is the validity of the forecasts in amplitude and in sign (detailed data will be published separately).

Institute of Applied Geophysics
Academy of Sciences of the USSR

Received
19 XII 1959

CITED LITERATURE

1. E. N. Blinova, *Dokl. Akad. Nauk SSSR*, **39**, No. 7 (1943).

2. E. N. Blinova, *Tellus*, **9**, No. 4 (1957).

3. S. A. Mashkovich, Ya. M. Kheifets, *Tr. Tsentral' n. inst. prognozov*, issue 60 (1957).

4. *Meteorological Bulletin*, Central Institute of Forecasts, IGY, July 1958, January 1959.

* The "steady regime" was introduced by the author in solving this problem in 1950; the solution immediately began to be used under operational conditions. Later an attempt was made to include in the treatment the influence of the initial temperatures (E. M. Dobryshman [2]). However, allowance for this influence did not improve the quality of the forecast.

Note: Figure translations are in progress. See original paper for figures.