THE THEORY OF THE "RETURN" LAYER

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Fig. 1

Figure 1: Fig. 1

Abstract

Full Text

HYDROMECHANICS

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THE THEORY OF THE "RETURN" LAYER

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As we showed in paper ([1]), the interaction of a free-molecular plasma flow with an external magnetic field leads to the phenomenon of "pressing away" of the flow, which consists in the fact that charged particles are elastically reflected from the surface of a certain cavity that includes the magnetic field distorted, in comparison with the initial one, together with the singularities causing it. This type of magnetic "pressing away" is produced by the interaction of corpuscular streams from the Sun with the magnetic field of the Earth, if the former have densities corresponding approximately to 10^2 particles per 1 cm^3, and a static temperature greater than $10^3\,^\circ$K.

In paper ([1]) it was indicated that the boundary of separation between the magnetic field and the plasma flow is a "return" layer, where the magnetic field increases from zero to a certain limiting value $H_{\max}$, and where, in the general case, there is an electric field; moreover, both the magnetic and the electric field are self-consistent, i.e., are caused by the motion of charged particles in the "return" layer.

Fig. 1

In the present work a theory is given of a relativistic "return" layer for the case when the velocity of motion of the plasma is perpendicular to it. Its equations are derived, and an investigation of the order of the terms is carried out. An exact solution of these equations is obtained in the case of nonrelativistic motion in the "return" layer of plasma particles with equal mass of charged particles; an approximate solution of the problem is also given for a real plasma, which is a mixture of protons and electrons, for the nonrelativistic case. The existence, in the latter case, of a new mechanism of electron acceleration is indicated.

The theory of the nonrelativistic "return" layer was first considered by Chapman and Ferraro ([2]); they obtained, in approximate form, its electromagnetic characteristics.

§ 1. Let us derive the equations for the "return" layer. Let the plane x, y (see Fig.

1) be the plane in which the motion occurs of plasma particles incident with velocity u_0, directed along the x-axis, on the "return" layer, where the plasma is dispersed in such a way that ions move to one side and electrons to the other; in the "return" layer and to the right of it the magnetic field **H**, different from zero, is directed perpendicular to the plane x, y and depends only on the coordinate x; the electric field, different from zero only in the "return" layer, is directed along the x-axis and is likewise a function only of x. Let the velocities of electrons and ions along the axes x and y be, respectively: u_e, u_i; v_e, v_i.

Maxwell' s equations for the electric and magnetic fields can be written in the form

$$\frac{dE}{dx} = 4\pi\rho; \qquad \frac{dH}{dx^*} = -\frac{4\pi}{c}j. \tag{1}$$

The charge density ρ and the current density j in the "returning" layer are expressed in the following way in terms of the velocities of the electrons and ions:

$$\rho = 2en_0u_0\left(\frac{1}{u_i} - \frac{1}{u_e}\right); \qquad j = 2en_0u_0\left(\frac{v_i}{u_i} - \frac{v_e}{u_e}\right) \tag{2}$$

(n_0 is the ion density in the incident stream; the charges of the electrons and ions are assumed equal in magnitude to e).

On the basis of (1) and (2)

$$E = 8\pi en_0u_0\left(\int\frac{dx}{u_i} - \int\frac{dx}{u_e}\right); \qquad H = \frac{8\pi en_0u_0}{c}\left(\int\frac{v_e}{u_e}\,dx - \int\frac{v_i}{u_i}\,dx\right); \tag{3}$$

the integrals are taken from the beginning of the "returning" layer on the left up to the current coordinate x. The expressions (3) can also be represented in the form

$$E = 8\pi en_0u_0(t_i - t_e); \qquad H = \frac{8\pi en_0u_0}{c}(y_e - y_i), \tag{4}$$

where t_e (t_i) is the time spent by an electron (ion) in traveling along its trajectory to the point with abscissa x; y_e, y_i are the ordinates acquired in this process.

The equations of motion of the electrons and ions will be written in the form

$$\begin{aligned}
\frac{dp_{xe}}{dt_e} &= -8\pi e^2 n_0 u_0 \left[(t_i - t_e) + \frac{1}{c^2}\frac{dy_e}{dt_e}(y_e - y_i)\right];\\
\frac{dp_{ye}}{dt_e} &= \frac{8\pi e^2 n_0 u_0}{c^2}\frac{dx}{dt_e}(y_e - y_i);\\
\frac{dp_{xi}}{dt_i} &= 8\pi e^2 n_0 u_0 \left[(t_i - t_e) + \frac{1}{c^2}\frac{dy_i}{dt_i}(y_e - y_i)\right];\\
\frac{dp_{yi}}{dt_i} &= -\frac{8\pi e^2 n_0 u_0}{c^2}\frac{dx}{dt_i}(y_e - y_i),
\end{aligned} \tag{5}$$

where $\mathbf{p}_e$ and $\mathbf{p}_i$ are the momenta of the electrons and ions.

§ 2. Let $m_e = m_i = m$; then $y_e = -y_i = y$, $t_e = t_i = t$, and if $u_0 \ll c$, then

$$x'' = -\alpha y' y; \qquad y'' = \alpha x' y \qquad \left(\alpha = \frac{16\pi e^2 n_0 u_0}{c^2 m}\right). \tag{6}$$

The transformation

$$\bar t = \frac{4eu_0}{c}\sqrt{\frac{\pi n_0}{m}}\, t; \qquad \bar y = \frac{4e}{c}\sqrt{\frac{\pi n_0}{m}}\, y; \qquad \bar x = \frac{4e}{c}\sqrt{\frac{\pi n_0}{m}}\, x \tag{7}$$

brings the system (6) to the form

$$\bar x'' = -\bar y' \bar y, \qquad \bar y'' = \bar x' \bar y. \tag{8}$$

Integrating first the first equation of this system, and then the second, we have

$$\bar x' = 1 - \frac{\bar y^2}{2}; \qquad \bar{y'}^2 = \bar y^2 - \frac{\bar y^4}{4} \tag{9}$$

(the conditions $\bar y = 0,\ \bar y' = 0,\ \bar x' = 1$ have been taken into account).

Integrating equations (9), we obtain

$$\begin{aligned}
\bar y &= \frac{2}{\operatorname{ch}\bar t} \qquad (-\infty \leqslant \bar t \leqslant -|\operatorname{Ar\,ch}\sqrt{2}|),\\
\bar x &= -\left|\operatorname{Ar\,ch}\frac{2}{\bar y}\right| + \sqrt{4 - \bar y^2} \qquad (0 \leqslant \bar y \leqslant \sqrt{2})
\end{aligned} \tag{10}$$

(the origin for $\bar x$ and $\bar t$ is chosen so that the point corresponding to $\bar x' = 0,\ \bar y = \sqrt{2}$ has the coordinates $\bar x_{\max} = -|\operatorname{Ar\,ch}\sqrt{2}| + \sqrt{2},\ \bar t_{\max} = -|\operatorname{Ar\,ch}\sqrt{2}|$).

From expression (10) it is seen that as $\bar{y} \to 0$, $\bar{y} \simeq e^{\bar{x}}$, i.e. the "return" layer has, strictly speaking, infinite thickness; however, the situation here is analogous to that which occurs in the hydrodynamics of the boundary layer: the actual thickness of the "returning" layer, owing to the exponential law of decrease of the quantity $\bar{y}$, amounts to several units of $\bar{x}$, i.e. is of the order

$$\frac{c}{4e}\sqrt{\frac{m}{\pi n_0}}.$$

§ 3. Let us now turn to the practically most interesting case, when $m_e/m_i \ll 1$; let us also suppose that the parameter $\omega = (u_0/c)^2 m_i/m_e$ is of order unity or less. We introduce the transformation

$$x = \bar{x}L; \qquad y_i = \bar{y}_i L\sqrt{\frac{m_e}{m_i}}; \qquad y_e = \bar{y}_e L\sqrt{\frac{m_i}{m_e}};$$

$$t_i = \bar{t}_i \frac{L}{u_0}; \qquad t_e = \bar{t}_e \frac{L}{u_0} \qquad \left(L^2 = \frac{m_e c^2}{8\pi e^2 n_0}\right). \tag{11}$$

Assuming that the barred quantities are of order unity (this will be clear from what follows), substituting expression (11) into the system of equations (5) and omitting terms which, relative to those retained, are of order m_e/m_i, we arrive at the system of equations:

$$\bar{t}_i - \bar{t}_e = \omega \frac{d^2\bar{x}}{d\bar{t}_i^2}; \qquad \frac{d^2\bar{y}_i}{d\bar{t}_i^2} = -\frac{d\bar{x}}{d\bar{t}_i}\,\bar{y}_e;$$

$$\frac{d^2\bar{x}}{d\bar{t}_i^2} = -\frac{d\bar{y}_e}{d\bar{t}_e}\,\bar{y}_e; \qquad \frac{d}{d\bar{t}_e}\frac{d\bar{y}_e/d\bar{t}_e}{\sqrt{1-\omega(d\bar{y}_e/d\bar{t}_e)^2}} = \frac{d\bar{x}}{d\bar{t}_e}\,\bar{y}_e. \tag{12}$$

Expressions (11) are the similarity law for the "returning" layer in the case $m_e \ll m_i$ and $\omega \lesssim 1$.

In this case the "returning" layer is characterized by the following features:

1) The currents in the "returning" layer are produced only by the motion of the electrons.
2) The principal component of the force acting on the ions in the direction of the x-axis is the electric field.
3) The force of action of the electric field on an electron is balanced by the x-component of the action on it of the magnetic field, so that the x-component of the inertial force for the electron may be neglected.

Fig. 2

Figure 2: Fig. 2

4) The case $\omega \sim 1$ means that $u_0 \ll c$, but, as is seen from expressions (11), the velocity of the electrons in the "returning" layer can reach velocities of the order of the speed of light, i.e. the electrons are accelerated in the "returning" layer.

§ 4. Let us now turn to the consideration of the "returning" layer of the Earth. Let in this case $u_0 \sim 10^8$ cm/sec, $n_0 \sim 10^2$ cm^{-3}, $m_e/m_i = 1/1860$, i.e. small; the quantity $\omega = 2.07 \cdot 10^{-2}$, i.e. also small; consequently, in equations (12) further simplifications are possible, associated with the smallness

quantities ω; we shall have $\bar{t}_e = \bar{t}_i = \bar{t}$, $\bar{y}_i = -\bar{y}_e = \bar{y}$, and equations (12) take the form

$$\bar{x}'' = -\bar{y}'\bar{y}; \qquad \bar{y}'' = \bar{x}'\bar{y}, \tag{13}$$

i.e., in a remarkable way they coincide with the previously considered system of equations (8) for the case $m_e = m_i$. Its solution will be, as before, expressions (10); the trajectory $\bar{y} = \bar{y}(\bar{x})$ is shown in Fig. 2. But if previously $y_e = -y_i$, now, as is seen from expressions (11), $y_e = -y_i m_i/m_e$. The thickness of the "return" layer is of the order

$$L^* = \sqrt{\frac{m_e c^2}{16\pi e^2 n_0}} = 2.6 \cdot 10^4 \text{ cm}. \tag{14}$$

Fig. 2

It is easy to obtain the relation

$$\frac{H_{\max}^2}{8\pi} = 2n_0 m_i u_0^2, \tag{15}$$

which is a particular case of relation (2) of paper [1].

From expression (15) it follows that the quantities of the Larmor radii of the electrons and ions will be, respectively:

$$r_e = L_i^* \sqrt{\frac{m_e}{m_i}}; \qquad r_i = L^* \sqrt{\frac{m_i}{m_e}}, \tag{16}$$

i.e., they differ in order of magnitude from the thickness of the "return" layer L^*.

As follows from expressions (11), $u_e \sim u_i \sim u_0$; $v_i \sim u_0\sqrt{m_e/m_i} \ll u_0$, $v_e \sim u_0\sqrt{m_i/m_e} \gg u_0$, i.e., the electrons in the "return" layer are accelerated owing to the formation of space charge.

The maximum electron velocity is attained at the point $\bar{x}' = 0$; $\bar{y} = \sqrt{2}$; it is equal to

$$V_{e\,\max} = u_0\sqrt{\frac{m_i}{m_e}} = 4.3 \cdot 10^9 \text{ cm/sec}, \tag{17}$$

which corresponds to an electron energy equal to 5.23 keV.

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[1] V. N. Zhigulev, DAN, **135**, No. 6 (1960). [2] L. Dungey, *Cosmical Electrodynamic*, Cambridge, 1956.

Note: Figure translations are in progress. See original paper for figures.