A METHOD FOR CALCULATING SHOCK WAVES

To find discontinuous solutions of the equations of gas dynamics

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Abstract

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A METHOD FOR CALCULATING SHOCK WAVES

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To find discontinuous solutions of the equations of gas dynamics

$$\frac{\partial V}{\partial t} = \frac{\partial u}{\partial x},$$

$$\frac{\partial u}{\partial t} = -\frac{\partial P}{\partial x}, \tag{1}$$

$$\frac{\partial E}{\partial t} = -P\frac{\partial V}{\partial t},$$

$$P = f(E, V),$$

where u is the velocity, P the pressure, V the specific volume, E the internal energy, and $f(E, V)$ an arbitrary function, difference methods of the so-called "through" computation ([1−3]) are widely used; these methods do not single out, on the grid of points covering the domain of integration of system (1), special points at which the functions u, P, V, E undergo a discontinuity.

In the present communication we propose a method for calculating discontinuous solutions of system (1) which does not single out the discontinuities specially and thus belongs to the methods of "through" computation, but which at the same time uses, for the computations, the Hugoniot conditions

$$\bar{V} - V_0 = -\frac{1}{w}(\bar{u} - u_0),$$

$$\bar{u} - u_0 = \frac{1}{w}(\bar{P} - P_0), \tag{2}$$

$$\bar{E} - E_0 = \frac{1}{2}(\bar{P} + P_0)(V_0 - \bar{V}),$$

which are valid only at discontinuities. Here $\bar{u}, \bar{P}, \bar{V}, \bar{E}$ characterize the state of the substance behind the front, while u_0, P_0, V_0, E_0 are those ahead of the shock-wave front, and w is the velocity of the shock wave.

We divide the domain of integration of system (1) by a grid into layers (intervals) of mass

$$h_{i+1/2} = x_{i+1} - x_i \quad (i = 0, 1, 2, \dots, N).$$

We shall determine u at the boundaries of the intervals (at points with integer indices), and the quantities P, V, E at the centers of the intervals (at points with half-integer indices). Each interval is then characterized by two values of the velocity u (at the right and left boundaries) and by the values P, ρ, E.

We divide all intervals into two classes. To the first class we assign intervals characterized by the condition

$$\frac{\Delta u}{\Delta x} = \frac{u_{i+1} - u_i}{x_{i+1} - x_i} \geq 0,$$

and to the second—those characterized by the condition

$$\frac{\Delta u}{\Delta x} = \frac{u_{i+1} - u_i}{x_{i+1} - x_i} < 0.$$

We shall call an approximate solution in intervals of the first class an **elementary rarefaction wave**, and in intervals of the second class an **elementary shock wave**.

In the case where $\Delta u/\Delta x < 0$, we shall regard the difference $u_{i+1}^{n+1} - u_i^{n+1}$ as a discontinuity of velocity on an elementary shock wave, and the values $P_{i+1/2}^n$, $V_{i+1/2}^n$, $E_{i+1/2}^n$ as the values of the corresponding quantities ahead of the front of the elementary shock wave in the interval $i + 1/2$.

Solving, for the elementary shock wave in the interval $i + 1/2$, system (2) with the known values $P_0 = P_{i+1/2}^n$, $V_0 = V_{i+1/2}^n$, $E_0 = E_{i+1/2}^n$, and with the quantity $\bar{u} - u_0 = \pm(u_{i+1}^{n+1} - u_i^{n+1})$, we determine $\bar{P}_{i+1/2}^{n+1}$, $\bar{V}_{i+1/2}^{n+1}$, $\bar{E}_{i+1/2}^{n+1}$. The value of the pressure $\bar{P}_{i+1/2}^{n+1}$ thus determined is subsequently used to compute $E_{i+1/2}^{n+1}$, $P_{i+1/2}^{n+1}$.

Finally, we arrive at the following difference equations:

$$u_i^{n+1} = u_i^n - \frac{\tau}{h}\left(\bar{P}_{i+1/2}^n - \bar{P}_{i-1/2}^n\right)$$

$$V_{i+1/2}^{n+1} = V_{i+1/2}^n + \frac{\tau}{h}\left(u_{i+1}^{n+1} - u_i^{n+1}\right),$$

$$E^{n+1}_{i+1/2} = E^{n}_{i+1/2} + \frac{1}{2}\left(\bar{P}^{n}_{i+1/2} + \bar{P}^{n+1}_{i+1/2}\right)\left(V^{n}_{i+1/2} - V^{n+1}_{i+1/2}\right),$$

$$P^{n+1}_{i+1/2} = f\left(E^{n+1}_{i+1/2}, V^{n+1}_{i+1/2}\right). \tag{3}$$

The quantity $\bar{P}^{n+1}_{i+1/2}$ is determined as follows:

1) If

$$\frac{u^{n+1}_{i+1} - u^{n+1}_{i}}{x_{i+1} - x_i} \geq 0,$$

then

$$\bar{P}^{n+1}_{i+1/2} = P^{n+1}_{i+1/2}.$$

2) If

$$\frac{u^{n+1}_{i+1} - u^{n+1}_{i}}{x_{i+1} - x_i} < 0,$$

then $\bar{P}^{n+1}_{i+1/2}$ is found as the solution of the system

$$\left(\bar{P}^{n+1}_{i+1/2} - P^{n}_{i+1/2}\right)\left(V^{n}_{i+1/2} - \bar{V}^{n+1}_{i+1/2}\right) - \left(u^{n+1}_{i+1} - u^{n+1}_{i}\right)^2 = 0,$$

$$\bar{E}^{n+1}_{i+1/2} - E^{n}_{i+1/2} - \frac{1}{2}\left(P^{n}_{i+1/2} + \bar{P}^{n+1}_{i+1/2}\right)\left(\bar{V}^{n+1}_{i+1/2} - V^{n}_{i+1/2}\right) = 0,$$

$$\bar{P}^{n+1}_{i+1/2} = f\left(\bar{E}^{n+1}_{i+1/2}, \bar{V}^{n+1}_{i+1/2}\right).$$

When the proposed method is used, any shock wave is automatically replaced by a finite number of elementary shock waves (a shock layer).

The trajectory of the shock wave being computed is determined as the trajectory of that elementary shock wave belonging to the shock layer on which the quantity $\bar{P} - P$ attains its maximum value, while the discontinuities u, P, V, E on the shock wave being computed are determined as the differences of the corresponding quantities taken at the right and left boundaries of the shock layer.

The computational process is stable under the condition

$$\tau \ll \frac{h}{c},$$

where c is the speed of sound.

With the aid of this method, numerous computations were carried out. Some of them were compared with exact solutions, and some with approximate solutions obtained by the method of characteristics. The agreement of the results was quite satisfactory.

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Note: Figure translations are in progress. See original paper for figures.